This yacht is propelled by a sail, a motor, or both.
Many yachts can be powered by the wind, by a gas engine, or both. A hybrid automobile can run on gasoline or electric power. These combinations are very powerful. Combining addition or subtraction with multiplication or division is powerful as well. You can use equations and expressions with mixed operations to solve many complex problems.

**Big Ideas**

- You can use the laws of arithmetic to solve algebraic equations and inequalities.
- Solving an equation or inequality means finding values for the variable or variables that make the equation or inequality a true statement.

**Unit Topics**

- The Distributive Property
- Like Terms
- Expressions with Mixed Operations
- Equations with Mixed Operations
- Error Analysis
- Inequalities
The Distributive Property

When you simplify an expression involving parentheses, the order of operations tells you to begin inside parentheses and work your way out. There is another way to simplify expressions that have parentheses.

The distributive property combines multiplication with either addition or subtraction.

**DISTRIBUTIVE PROPERTY**

For all numbers \( a, b, \) and \( c \):

\[
\begin{align*}
\text{If } a(b + c) &= ab + ac \\
\text{and } a(b - c) &= ab - ac
\end{align*}
\]

**Examples**

\[
\begin{align*}
3(5 + 2) &= 3 \cdot 5 + 3 \cdot 2 \\
6(4 - 10) &= 6 \cdot 4 - 6 \cdot 10
\end{align*}
\]

**Verifying the Distributive Property**

You can use the order of operations to verify the distributive property.

**Example 1** Verify the distributive property in each case.

**A** \( 3(5 + 2) = 3 \cdot 5 + 3 \cdot 2 \)

**Solution**

\[
\begin{align*}
3(5 + 2) &= 3 \cdot 5 + 3 \cdot 2 \\
3(7) &= 15 + 6 \\
21 &= 21
\end{align*}
\]

On the left side, add inside parentheses first. On the right side, use the distributive property to multiply first.

**B** \( 6(4 - 10) = 6 \cdot 4 - 6 \cdot 10 \)

**Solution**

\[
\begin{align*}
6(4 - 10) &= 6 \cdot 4 - 6 \cdot 10 \\
6(-6) &= 24 - 60 \\
-36 &= -36
\end{align*}
\]

On the left side, subtract inside parentheses first. On the right side, use the distributive property to multiply first.

**TIP**

When you use the distributive property, you have **distributed** the factor through the terms.
Using the Distributive Property to Rewrite Expressions

You can use the distributive property to change an addition expression without parentheses to one with parentheses (and vice versa).

**Example 2**  Rewrite each expression without grouping symbols.

A  \(2(5 + 7)\)

**Solution**

\[2(5 + 7) = 2 \cdot 5 + 2 \cdot 7\]

B  \(-3(4 + x)\)

**Solution**

\[-3(4 + x) = -3 \cdot 4 + (-3) \cdot x\]

**Example 3**  Rewrite each expression with grouping symbols.

A  \(2 \cdot 3 + 2 \cdot 11\)

**Solution**

\[2 \cdot 3 + 2 \cdot 11 = 2(3 + 11)\]

B  \(8 \cdot 1 + 8 \cdot (-7)\)

**Solution**

\[8 \cdot 1 + 8 \cdot (-7) = 8(1 + (-7)) = 8(1 - 7)\] 

Simplify \(1 + (-7)\) by writing \(1 - 7\).

C  \(5r - 5 \cdot 12\)

**Solution**

\[5r - 5 \cdot 12 = 5(r - 12)\]

Using the Distributive Property to Evaluate Expressions

Using the distributive property to combine multiplication with either addition or subtraction gives you a powerful tool for evaluating certain types of expressions.

**Example 4**  Use properties and mental math to evaluate each expression.

A  \(6 \cdot 74\)

**Solution**

\[6 \cdot 74 = 6(70 + 4)\] 

Write 74 as a sum.

\[= 6 \cdot 70 + 6 \cdot 4\] 

Apply the distributive property.

\[= 420 + 24\] 

Evaluate the expression.

\[= 444\]
8 · 99

Solution
8 · 99 = 8(100 − 1) Write 99 as a difference.
     = 8 · 100 − 8 · 1 Apply the distributive property.
     = 800 − 8 Evaluate the expression.
     = 792

7 · 67 + 7 · 3

Solution
7 · 67 + 7 · 3 = 7(67 + 3) Apply the distributive property.
     = 7(70) The sum 70 is easier to multiply.
     = 490

Solving Equations by Recognizing the Distributive Property

Example 5
Solve each equation.

A 3 · 5 + 3 · a = 3(5 + 7)

Solution By the distributive property, you know that
3 · 5 + 3 · 7 = 3(5 + 7), so a = 7.

B b(1.2 + 5.4) = 8 · 1.2 + 8 · 5.4

Solution By the distributive property, you know that
8(1.2 + 5.4) = 8 · 1.2 + 8 · 5.4, so b = 8.

Application: Perimeter

Example 6 Find the perimeter of the rectangle.

Solution
P = 2l + 2w Write the formula for perimeter of a rectangle.
   = 2 · 6.75 + 2 · 3.25 Substitute 6.75 for length l and 3.25 for width w.
   = 2(6.75 + 3.25) Apply the distributive property and then add the decimals to get 10, which is easy to multiply.
   = 2(10)
   = 20

The perimeter is 20 meters.
Problem Set

Verify the distributive property in each case.

1. \[2(3 + 6) = 2 \cdot 3 + 2 \cdot 6\]
2. \[10(5 + 1) = 10 \cdot 5 + 10 \cdot 1\]
3. \[-1(-3 + 2) = -1 \cdot (-3) + (-1) \cdot 2\]
4. \[4(6 - 1) = 4 \cdot 6 - 4 \cdot 1\]
5. \[2(8 - 9) = 2 \cdot 8 - 2 \cdot 9\]
6. \[-6(1 - 3) = -6 \cdot 1 - (-6) \cdot 3\]

Rewrite each expression without grouping symbols.

7. \[5(5 + 1)\]
8. \[2(6 + 6)\]
9. \[4(v + 7)\]
10. \[-20(2 + 8)\]
11. \[-9(1 + y)\]
12. \[4(-8 + w)\]

Rewrite each expression with grouping symbols.

13. \[3 \cdot 6 + 3 \cdot 9\]
14. \[7 \cdot 5 + 7 \cdot (-6)\]
15. \[4 \cdot 2 - 4 \cdot 11\]
16. \[3x - 3 \cdot 7\]
17. \[5 \cdot 8 - 5s\]
18. \[6n - 6d\]
19. \[-2x - 2y\]
20. \[10a - 10 \cdot 1\]

Use properties and mental math to evaluate each expression.

21. \[2 \cdot 65\]
22. \[5 \cdot 37\]
23. \[6 \cdot 99\]
24. \[5 \cdot 98\]
25. \[8 \cdot 102\]
26. \[9 \cdot 34 + 9 \cdot 6\]
27. \[8 \cdot 87 + 8 \cdot 13\]
28. \[7 \cdot 248 - 7 \cdot 48\]
29. \[4.58 \cdot 101\]
30. \[12.05 \cdot 86 - 12.05 \cdot 75\]

Use the distributive property to solve each equation.

31. \[2 \cdot 6 + 2 \cdot x = 2(6 + 3)\]
32. \[5 \cdot a + 5 \cdot 7 = 5(-1 + 7)\]
33. \[n(4 + 16) = 7 \cdot 4 + 7 \cdot 16\]
34. \[-2(x + 1) = -2 \cdot 4 + (-2) \cdot 1\]
35. \[10(-5 + s) = 10 \cdot (-5) + 10 \cdot (-1)\]
36. \[a(1.5 + 1.5) = -3 \cdot 1.5 + (-3) \cdot 1.5\]

Use the distributive property to solve each problem. Show your work.

37. Find the perimeter of the rectangle.

38. Find the perimeter of the equilateral hexagon.

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39. The diagram represents a parcel of land formed by two rectangular regions. Find the total area.

![Diagram of land parcel]

40. Find the total cost of each set of purchases.
   A. 5 gallons of milk at $3.95 per gallon
   B. 9 square yards of carpet at $29.99 per square yard
   C. 6 boxes of popcorn at $1.85 per box
   D. 4 bars of soap at $1.67 each and 4 tubes of toothpaste at $2.33 each
   E. 15 towels at $11.25 each and 15 washcloths at $3.75 each

41. Challenge Find the total cost of each set of purchases. Round each answer to the nearest cent.
   A. 8 gallons of gasoline at $2.999 per gallon
   B. 20 ounces of cereal at $0.241 per ounce
Like Terms

To simplify some variable expressions, you need to identify and combine like terms.

Identifying Like Terms

A sum or difference expression is made up of terms. Like terms have the same variable part or parts. Numbers are considered to be like terms; they can be called numerical terms. Terms are made up of factors that are multiplied together.

Example 1  Identify the like terms.

A  5x, 5y, −2x, 5, −11

Solution

{5x, −2x}, {5, −11}, 5y The terms 5x and −2x have the same variable. The terms 5 and −11 are numerical terms.

The terms 5x and −2x form one pair of like terms; and the terms 5 and −11 form another pair of like terms. ■

B −3a, 2b, 6ab, 1, a

Solution  The term 6ab is the only term that has both variables a and b, so it is not like any of the other terms.

{−3a, a}, 2b, 6ab, 1 The terms −3a and a have the same variable.

The terms −3a and a are like terms. ■

C 6, 4p, 6p², −p, 3pq

Solution

6, {4p, −p}, 6p², 3pq The terms 4p and −p have the same variables.

The terms 4p and −p are like terms. ■

Using the Distributive Property to Combine Like Terms

The distributive property states that \( a(b + c) = ab + ac \), which can also be written \( ba + ca = (b + c)a \). So, if you have an expression like \( 5x - 2x \), you can use the distributive property to write it as \( (5 - 2)x \) and simplify it to \( 3x \). This is how you combine like terms.
Example 2  Combine like terms.

A  \(5x + 2x\)

Solution

\[5x + 2x = (5 + 2)x\]  \[\text{Apply the distributive property } ba + ca = (b + c)a.\]

\[= 7x\]  \[\text{Add inside the parentheses.} \]

B  \(3y - y + 6y\)

Solution

\[3y - y + 6y = 3y - 1y + 6y\]  \[\text{Write } -y \text{ as } -1y \text{ so that it has a numerical factor.}\]

\[= (3 - 1 + 6)y\]  \[\text{Apply the distributive property.}\]

\[= 8y\]  \[\text{Simplify inside the parentheses.} \]

Identifying and Combining Like Terms

The numerical factor in a term is called a **coefficient**. To combine like terms, you only need to add or subtract the coefficients.

**COMBINING LIKE TERMS**

To combine like terms, combine their coefficients. Keep the variable part the same.

<table>
<thead>
<tr>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4x + 5x = 9x)</td>
</tr>
<tr>
<td>(3ab + 2ab - 1ab = 4ab)</td>
</tr>
</tbody>
</table>

**REMEMBER**

Combining like terms is based on the distributive property.

\[4x + 5x = (4 + 5)x = 9x\]

Example 3  Combine the like terms in each expression.

A  \(2a + 3 - 5a + 8b + 3ab\)

Solution

\[2a + 3 - 5a + 8b + 3ab = 2a - 5a + 3 + 8b + 3ab\]  \[\text{Rewrite so that like terms are together.}\]

\[= -3a + 3 + 8b + 3ab\]  \[\text{Combine like terms.} \]

B  \(3s + rs - 4r - 6rs - 8rs\)

Solution

\[3s + rs - 4r - 6rs - 8rs = 3s - 4r + 1rs - 6rs - 8rs\]  \[\text{Rewrite } rs \text{ as } 1rs. \text{ Write like terms together.}\]

\[= 3s - 4r - 13rs\]  \[\text{Combine like terms.} \]

Application: Sales Tax

Example 4  The sales tax rate in a certain state is 5%. Write a simplified expression for the total cost of an item that has a price of \(x\) dollars.

(continued)
Solution  The decimal equivalent of 5% is 0.05. When you pay 5% of \( x \) dollars in sales tax, you pay 0.05\( x \) dollars in sales tax.  

Total cost = price of item + sales tax  
= \( x \) + 0.05\( x \)  
= 1.00\( x \) + 0.05\( x \)  
Write \( x \) as 1.00\( x \).  
= 1.05\( x \)  
Combine like terms.  
The total amount paid is 1.05\( x \) dollars. ■

Problem Set

Identify the like terms.

1. 4\( x \), 4\( y \), 3\( y \), 5\( x \), −11
2. 4\( a \), −2\( b \), \( a \), 1, −1
3. 3\( c \), 3\( d \), 6\( c \), 6, 4\( a \)
4. −\( x \), \( y \), 6\( x \), −8\( x \), −8, −8\( z \)
5. 4\( ab \), −2\( b \), 4\( a \), 7, −5\( ab \), −5
6. 12\( c \), \( d \), −\( c \), 12, 4\( d \)
7. −5\( u \), 9\( v \), 6\( uv \), 2\( v \), −\( uv \), −5

Combine like terms.

15. 4\( a \) + 3\( a \)
16. 7\( x \) + 7\( x \)
17. −3\( c \) + 5\( c \)
18. \( \frac{1}{4} x - \frac{1}{9} x \)
19. −5\( x \) − 2.2\( x \) + \( x \)

Combine the like terms in each expression.

25. 8\( a \) + 5\( b \) − 2\( a \)
26. \( x \) + 4\( x \) − 6\( y \) + 5
27. 3\( t \) + 4\( t \) − \( t \) + 5
28. 5\( xy \) − 3\( x \) + 6\( y \) − \( xy \) + \( y \)
29. \( a \) − \( a \) + 6\( b \) − \( b \) − 5\( b \) + 6
30. 3\( ax \) + 3\( x \) − 5\( ax \) − \( x \) − 5\( a \) + 10\( ax \)
31. \( z \) + 4\( w \) − (−3\( w \)) + 12\( z \) − \( z \)
32. 3.5\( x \) + 4\( x \) − \( y \) + 2.5\( y \) − 1

33. \( r \) + \( rs \) − 5\( rs \) − 5\( rs \) + 2\( r \) − 6
34. 4 − \( cd \) − 2\( cd \) − 3\( cd \) + 4 − \( c \) + \( d \)
35. 5(\( x \) + 3) − 2\( x \)
36. \( a \) + 3(\( a \) − 1) − 5\( a \)
37. −6\( x \) + 2.5(\( x \) − 1) − 5\( x \)
38. \( x \) + \( \frac{3}{4} x \) − \( y \) + 5\( y \) + 3
39. 5\( b \) + 3(2\( b \) + 1) − 5\( ab \) − 3

*40. Challenge 2.4(2\( x \) − 5\( r \)) + 3(1.1\( s \) + \( r \)) − 2\( x \)
Solve each problem.

41. Write a simplified expression for the total cost of each purchase described. Show how you combine like terms to obtain your simplified expression.
   A. The price of an item is $d$ dollars and the sales tax rate is 6%.
   B. The price of a meal is $m$ dollars and an 18% tip is added.
   C. The price of a hotel room is $r$ dollars and there is a hotel tax of 4.5%.

42. Write a simplified expression for the perimeter of each polygon.
   A. Rectangle
      \[ 5x \]
      \[ 3x \]
      \[ \text{Perimeter} = 2(5x) + 2(3x) \]
   B. Regular pentagon
      \[ 2x + 3 \]
      \[ \text{Perimeter} = 5(2x + 3) \]
   C. Two rectangles combined
      \[ 3x - 6 \]
      \[ 2x \]
      \[ 3x - 5 \]
      \[ x \]
      \[ \text{Perimeter} = (3x - 6) + (2x) + (3x - 5) + x \]

43. Challenge Coach Dixon is ordering T-shirts for his players. Each T-shirt costs $8.85. There is a $35 setup fee for silk-screening and a screening charge of $2.15 per shirt.
   A. Write a simplified expression for the total cost of $x$ T-shirts. Show how you combine like terms to obtain your simplified expression.
   B. Find the total cost of 12 T-shirts. Show how you evaluate your expression to obtain your answer.
   C. The T-shirt company offers a 10% discount per shirt for orders of at least 20 shirts. (The discount is applied to both the cost of the shirt and the screening charge.) Explain why 20 shirts cost less than 19 shirts.

44. Challenge The diagram consists of a circle with radius $r$ and a square. The circle is tangent to each side of the square, which means that the circle touches each side of the square in exactly one point. Write a simplified expression for the sum of the circumference of the circle and perimeter of the square. (Hint: The formula for the circumference of a circle is $C = 2\pi r$.)
Expressions with Mixed Operations

To simplify an expression that contains negative numbers, fractions, or decimals, use the **order of operations**.

**ORDER OF OPERATIONS**

- **Step 1** Perform operations within grouping symbols.
- **Step 2** Multiply and divide from left to right.
- **Step 3** Add and subtract from left to right.

**Simplifying Numerical Expressions**

**Example 1** Simplify.

**A** \( 5 \cdot 2 - 6(-2) \)

**Solution**

\[
5 \cdot 2 - 6(-2) = 10 - 6(-2) \quad \text{Multiply.}
\]
\[
= 10 - (-12) \quad \text{Multiply.}
\]
\[
= 10 + 12 \quad \text{To subtract \(-12\), add 12.}
\]
\[
= 22 \quad \blacksquare
\]

**B** \(-5 + 60 \div 5 - 2(-3)\)

**Solution**

\[
-5 + 60 \div 5 - 2(-3) = -5 + 12 - 2(-3) \quad \text{Divide.}
\]
\[
= -5 + 12 - (-6) \quad \text{Multiply.}
\]
\[
= -5 + 12 + 6 \quad \text{To subtract \(-6\), add 6.}
\]
\[
= 7 + 6 \quad \text{Add from left to right.}
\]
\[
= 13 \quad \blacksquare
\]

**C** \(-3 - 28 \div (3 + 1)\)

**Solution**

\[
-3 - 28 \div (3 + 1) = -3 - 28 \div 4 \quad \text{Add inside the grouping symbols.}
\]
\[
= -3 - 7 \quad \text{Divide.}
\]
\[
= -10 \quad \text{Subtract from left to right.} \quad \blacksquare
\]
**Expressions with Mixed Operations**

**Solution**

\[
\frac{2 \cdot (7 - 15)}{5 - 10 + 3}
\]

\[
= \frac{2 \cdot (-8)}{5 - 10 + 3}
\]

In the numerator, subtract inside the parentheses.

\[
= \frac{-16}{2}
\]

In the numerator, multiply. In the denominator, subtract and add from left to right.

\[
= 8
\]

Divide.

---

**Evaluating Algebraic Expressions**

Variables in an algebraic expression can stand for any values. When you need a specific value for the expression, you just need to replace the variables with specific values. To evaluate an algebraic expression, substitute values for the variables and then simplify the resulting numerical expression.

**Example 2**

**A** Evaluate \(7n + 5\) when \(n = -8\).

**Solution**

\[
7n + 5 = 7 \cdot (-8) + 5
\]

Substitute \(-8\) for \(n\).

\[
= -56 + 5
\]

Multiply.

\[
= -51
\]

Add.

**B** Evaluate \(\frac{c - 5d}{10(2d + 1)}\) when \(c = 195\) and \(d = -1\).

**Solution**

\[
\frac{c - 5d}{10(2d + 1)} = \frac{195 - 5 \cdot (-1)}{10(2 \cdot (-1) + 1)}
\]

Substitute 195 for \(c\) and \(-1\) for \(d\).

\[
= \frac{195 - 5}{10(-2 + 1)}
\]

Multiply \(5 \cdot (-1)\) in the numerator. Add \(2 \cdot (-1)\) in the denominator.

\[
= \frac{195 + 5}{10(-1)}
\]

To subtract \(-5\), add 5 in the numerator. Add \(-2 + 1\) in the denominator.

\[
= \frac{200}{-10}
\]

Add in the numerator. Multiply in the denominator.

\[
= -20
\]

Divide.

---

**REMEMBER**

A fraction bar represents division. It is also a grouping symbol.
Simplifying Expressions with Decimals and Fractions

**Example 3**  Simplify.

\[-2 \cdot 1.5 + 3 \cdot (-2.15) \frac{0.5}{0.5}\]

**Solution**

\[-2 \cdot 1.5 + 3 \cdot (-2.15) \frac{0.5}{0.5} = \frac{-3.0 + (-6.45)}{0.5} \quad \text{Multiply in the numerator.} \]

\[= \frac{-9.45}{0.5} \quad \text{Add in the numerator.} \]

\[= -18.9 \quad \text{Divide.} \]

\[-3 \cdot \frac{2}{5} + \frac{1}{2} \cdot (-6)\]

**Solution**

\[-3 \cdot \frac{2}{5} + \frac{1}{2} \cdot (-6) = \frac{3}{1} \cdot \frac{2}{5} + \left(\frac{1}{2}\right) \cdot \left(\frac{6}{1}\right) \quad \text{Write the integers as fractions.} \]

\[= \frac{6}{5} + (-3) \quad \text{Multiply the fractions.} \]

\[= \frac{6}{5} + \left(-\frac{15}{5}\right) \quad \text{Write -3 as a fraction with the denominator 5.} \]

\[= \frac{21}{5} \quad \text{Add.} \]

**REMEMBER**

You can write an improper fraction as a mixed number.

\[-\frac{21}{5} = -4\frac{1}{5}\]

**Application: Temperature**

**Example 4**  The formula to convert a temperature from degrees Celsius to Fahrenheit is \(F = \frac{9}{5}C + 32\). What is the Fahrenheit equivalent of \(-12\) degrees Celsius?

**Solution**  Use the formula to convert degrees Celsius to degrees Fahrenheit.

\[F = \frac{9}{5}C + 32 \quad \text{Write the formula.} \]

\[= \frac{9}{5} \cdot (-12) + 32 \quad \text{Substitute -12 for C.} \]

\[= \frac{9}{5} \cdot \left(-\frac{12}{1}\right) + 32 \quad \text{Write -12 as a fraction.} \]

\[= -\frac{108}{5} + 32 \quad \text{Multiply the fractions.} \]

\[= -21.6 + 32 \quad \text{Divide.} \]

\[= 10.4 \quad \text{Add.} \]

\(-12\)°C is equivalent to 10.4°F.
Problem Set

Simplify.
1. \(3 \cdot 5 + 6 \cdot (-1)\)
2. \(-6 \cdot 5 - 8 \cdot 10\)
3. \(10 \cdot (-3) - 3 \cdot (-4)\)
4. \(-6 \cdot (-1) + 6 \cdot 1 - 6 \cdot 2\)
5. \(-3 + 80 \div 8 - 5 \cdot (-1)\)
6. \(1 - 6 \cdot 3 + 5 \cdot 2\)
7. \(12 \div 6 - 10 \cdot 5 - 10 \cdot (-3)\)
8. \(7 + 2(3 + 7) - 5 \cdot (-3)\)
9. \(\frac{8}{3} + 10 \cdot 2\)
10. \(-5 - (-24) \div 3 + 11\)

Evaluate the expression, using the given value(s) of the variable(s).
11. \(20 - (-24 - 3) + 16 \div 4\)
12. \(-1 + 5(100 - 95) + 4 \cdot (-2)\)
13. \((62 - 12) - (4 \cdot 6)\)
14. \(\frac{23 + 5 \cdot (8 - 10)}{2 - 3 - 5}\)
15. \(2 - 8 \cdot 2 + 20 \div 4\)
16. Challenge \(2 \cdot 12 + 5 \cdot 2 - (1 + 20) + 40 \div (-2)\)
17. Challenge \(\frac{8 - 13}{2 + 3 \cdot (8 - 7)} - 2 + 16 \cdot (14 - 19)\)
18. Challenge \(16 - \frac{18}{9} \cdot 5\)

19. \(5a + 10\) when \(a = -3\)
20. \(-2c - 4\) when \(c = 5\)
21. \(22 - 5x - 3x\) when \(x = 6\)
22. \(-6a + a - 3b\) when \(a = -5\) and \(b = -1\)
23. \(r \div 3 - 10r + 3(r + 2)\) when \(r = 9\)
24. \(10 + 4(x - 6) - 5(x + 2)\) when \(x = 0\)
25. \(\frac{n}{2} + \frac{6n}{2} + n\) when \(n = 10\)
26. \(\frac{x - 2y}{5(3y + 5)}\) when \(x = 1\) and \(y = -2\)

31. Challenge \(10 - \frac{n}{2} \cdot \frac{n}{4} + n\) when \(n = 2.4\)
32. Challenge \(\frac{-4a + b}{a - b}\) when \(a = 2.5\) and \(b = 5\)

Simplify.
33. \(12 \cdot 1.2 + 3 \cdot (-0.6)\)
34. \(-4 \cdot \frac{1}{8} + 6 \cdot \frac{2}{3}\)
35. \(\frac{16.2 - 16}{16}\)
36. \(\frac{0.25 + 1.75}{40 \cdot 0.2}\)
37. \(5.6 + 1.05 \div 4.4\)
38. \(\frac{1}{4} \cdot 8 \div \frac{3}{4} \div \frac{4}{9}\)
39. \(0.25 \cdot (-4) \cdot (-0.6) \div -6 \cdot 0.02\)
40. Challenge \(\frac{-6 \cdot 2.5 + 2 \cdot (-1.4)}{0.4}\)
Solve each problem. Show your work.

41. The formula to convert degrees Celsius to degrees Fahrenheit is \( F = \frac{9}{5}C + 32 \). The formula to convert degrees Fahrenheit to degrees Celsius is \( C = \frac{5}{9}(F - 32) \).

A. What is the Fahrenheit equivalent of 5 degrees Celsius?
B. What is the Celsius equivalent of 5 degrees Fahrenheit?
C. What is the Celsius equivalent of −40 degrees Fahrenheit?

*42. Challenge* The formula for the area \( A \) of a trapezoid is \( A = \frac{1}{2}(b_1 + b_2)h \), where \( b_1 \) and \( b_2 \) represent the lengths of the parallel bases and \( h \) represents the height. Find the area of the trapezoid.

![Diagram of a trapezoid with dimensions: base 1 = 5.6 m, base 2 = 8.6 m, height = 4 m]
Equations with Mixed Operations

To solve equations with mixed operations, you need to understand variable terms and indicated operations.

In the equation $5 + 2x = 21$, the variable term is $2x$ and the indicated operations are as follows: The variable is multiplied by 2 and then 5 is added.

In the equation $\frac{t}{3} - 2 = 1$, the variable term is $\frac{t}{3}$ and the indicated operations are as follows: The variable is divided by 3 and then 2 is subtracted.

Solving Simple Equations

To solve an equation, you need to isolate the variable on one side. In $5 + 2x = 21$, subtract 5 from both sides to isolate the terms. The result is $2x = 16$. Similarly, in $\frac{t}{3} - 2 = 1$, add 2 to both sides. The result is then $\frac{t}{3} = 3$.

If an equation has only one variable term and at most one numerical term on each side, use the method described below to isolate the variable. The idea is to peel back the layers one at a time.

SOLVING SIMPLE EQUATIONS IN ONE VARIABLE

Undo indicated operations in reverse order.

Example 1  Solve and check. $3x - 18 = 39$

Solution  Think: The variable is multiplied by 3, and then 18 is subtracted. To isolate the variable, add 18 and then divide by 3.

$3x - 18 = 39$

$3x - 18 + 18 = 39 + 18$  Add 18 to both sides to undo the subtraction.

$3x = 57$

$\frac{3x}{3} = \frac{57}{3}$  Divide both sides by 3 to undo the multiplication.

$x = 19$

(continued)
Check

\[ 3x - 18 = 39 \]

Start with the original equation.

\[ 3 \cdot 19 - 18 = 39 \]

Substitute 19 for \( x \).

\[ 57 - 18 = 39 \]

Multiply.

\[ 39 = 39 \checkmark \]

Example 2

Solve.

A \[ 10 + \frac{a}{2} = 2 \]

Solution

Think: The variable is divided by 2, and then 10 is added. To isolate the variable, subtract 10 and then multiply by 2.

\[ 10 + \frac{a}{2} = 2 \]

\[ 10 - 10 + \frac{a}{2} = 2 - 10 \]

Subtract 10 from both sides to undo the addition.

\[ \frac{a}{2} = -8 \]

\[ 2 \cdot \frac{a}{2} = 2 \cdot (-8) \]

Multiply both sides by 2 to undo the division.

\[ a = -16 \]

B \[ -6 = \frac{2}{3}c + 1 \]

Solution

Think: The variable is multiplied by \( \frac{2}{3} \), and then 1

\[ -6 = \frac{2}{3}c + 1 \]

\[ -6 - 1 = \frac{2}{3}c + 1 - 1 \]

Subtract 1 from both sides to undo the addition.

\[ -7 = \frac{2}{3}c \]

\[ \frac{3}{2} \cdot (-7) = \frac{3}{2} \cdot \frac{2}{3}c \]

Multiply both sides by \( \frac{3}{2} \) to undo the multiplication.

\[ \frac{-21}{2} = 1 \cdot c \]

\[ \frac{-21}{2} = c \]

REMEMBER

The solution in Example 1 is 19, not 39.

TIP

In Example 2A, you could start by multiplying by 2, and then you would subtract 20 from each side.

TIP

Check all solutions. Substitute the solution into the original equation and verify that it makes a true statement.

TIP

In Example 2B, you could start by multiplying by \( \frac{3}{2} \), and then you would subtract \( \frac{3}{2} \) from each side. In general, undoing subtraction or addition first makes the numbers neater.
\[
\frac{x + 1}{3} = -5
\]

**Solution**  
*Think:* First 1 is added to the variable, and then the result is divided by 3. To isolate the variable, multiply by 3, and then subtract 1.

\[
\frac{x + 1}{3} = -5 \\
3 \cdot \frac{x + 1}{3} = 3 \cdot (-5) \quad \text{Multiply both sides by 3 to undo the division.} \\
x + 1 = -15 \\
x + 1 - 1 = -15 - 1 \quad \text{Subtract 1 from both sides to undo the addition.} \\
x = -16 \quad \blacksquare
\]

**THINK ABOUT IT**  
The equation in Example 2C can be written as \(\frac{1}{3}(x + 1) = -5\). You could solve it the same way as shown here.

---

## Combining Like Terms to Solve Equations

To solve some equations, it helps to simplify expressions by combining like terms. You don’t absolutely have to simplify first, but a simpler equation reduces your risk of making mistakes.

### Example 3  
**Solve.**

**A**  
\[5x + 5 - 8x - 4 = -29\]

**Solution**

\[
5x + 5 - 8x - 4 = -29 \\
-3x + 1 = -29 \\
-3x + 1 - 1 = -29 - 1 \quad \text{Subtract 1 from both sides to undo the addition.} \\
-3x = -30 \\
\frac{-3x}{-3} = \frac{-30}{-3} \quad \text{Divide both sides by -3 to undo the multiplication.} \\
x = 10 \quad \blacksquare
\]

**B**  
\[5(n - 2) - n = 14.8\]

**Solution**

\[
5(n - 2) - n = 14.8 \\
5n - 10 - n = 14.8 \quad \text{Apply the distributive property to remove parentheses.} \\
4n - 10 = 14.8 \quad \text{Combine like terms.} \\
4n - 10 + 10 = 14.8 + 10 \quad \text{Add 10 to both sides to undo the subtraction.} \\
4n = 24.8 \\
\frac{4n}{4} = \frac{24.8}{4} \quad \text{Divide both sides by 4 to undo the multiplication.} \\
n = 6.2 \quad \blacksquare
\]
Solving Equations that Have the Variable on Both Sides

If the variable appears on both sides of the equation, add or subtract a variable term so that the variable appears on only one side.

**Example 4**  Solve. \(3x + 1 = 5x - 7\)

**Solution**

\[
\begin{align*}
3x + 1 &= 5x - 7 \\
3x - 3x + 1 &= 5x - 3x - 7 \\
1 &= 2x - 7 \\
1 + 7 &= 2x - 7 + 7 \\
8 &= 2x \\
\frac{8}{2} &= \frac{2x}{2} \\
4 &= x
\end{align*}
\]

Subtract 3x from both sides so that the variable appears on only one side.

Add 7 to both sides to undo the subtraction.

Divide both sides by 2 to undo the multiplication.

**Application: Number Problem**

**Example 5**  Three more than twice a number is 31. What is the number?

**Solution**  Let \(n\) represent the number.

Three more than twice the number is 31.

\[
\begin{align*}
3 + 2n &= 31 \\
2n &= 28 \\
\frac{2n}{2} &= \frac{28}{2} \\
n &= 14
\end{align*}
\]

Write the equation.

Subtract 3 from both sides.

Divide by 2.

The number is 14. ■

**Application: Simple Interest**

**Example 6**  Andy deposited some money to open a bank account that paid 8% simple interest. He made no other deposits or withdrawals. At the end of one year, the balance was $270. How much did Andy deposit?

**Solution**  Let \(x\) represent the amount Andy deposited.

\[
\begin{align*}
\text{amount deposited} + \text{8% simple interest} &= \text{was} \quad \text{\$270} \\
x + 0.08x &= 270 \\
1.08x &= 270 \\
\frac{1.08x}{1.08} &= \frac{270}{1.08} \\
x &= 250
\end{align*}
\]

Write the equation.

Write \(x\) as 1.00\(x\).

Combine like terms.

Solve the equation.

Andy deposited $250. ■
Problem Set

Solve and check.

1. \(2x - 12 = 14\)
2. \(5x + 16 = 56\)
3. \(12.5 + 3x = 20\)
4. \(18 - 6.3x = 49.5\)
5. \(-1 + 12x = 29\)
6. \(10 - 1.4x = 10.84\)
7. \(-12x + 14 = -34\)
8. \(-24x - 15 = -27\)
9. \(6 + \frac{x}{3} = 1\)
10. \(-4 + \frac{a}{5} = 8\)
11. \(-1 = \frac{x}{9} - 2\)
12. \(-3 = \frac{3}{4}r + 2\)
13. \(\frac{2}{9}x - 7 = 9\)

Solve.

9. \(6 + \frac{x}{3} = 1\)
10. \(-4 + \frac{a}{5} = 8\)
11. \(-1 = \frac{x}{9} - 2\)
12. \(-3 = \frac{3}{4}r + 2\)
13. \(\frac{2}{9}x - 7 = 9\)
14. \(6 = \frac{9}{10}a - 1\)
15. \(\frac{x + 3}{5} = -1\)
16. \(44 = \frac{n - 6}{2}\)
17. \(-1 = -\frac{x + 1}{5}\)
18. Challenge \(\frac{3 - x}{9} = \frac{1}{4}\)

Solve.

19. \(2x + 6 - 7x - 10 = 16\)
20. \(-x + 11 - 2x - 4x = -24\)
21. \(0 = 14 - n + 4n - 10\)
22. \(-15 = 13 - 6e + c + 14.5\)
23. \(6(x - 1) - 2x = 38\)
24. \(-9(r - 1) + r = 20\)
25. \(4(a - 6) - a = 42.3\)
26. Challenge \(4(a - 11) - 7(2a + 1) = -2.28\)

Solve.

27. \(6b + 27 = -4b - 3\)
28. \(6x + 2 = 10x - 7\)
29. \(4(x + 5) = 3(x + 1) - 6\)
30. Challenge \(-2(5 - t) - t = 4(2t + 1) - 6\)

*31. Challenge \(s - 0.5(2 - s) = 7.3 = 4.5(2s + 10) - s\)

*32. Challenge \(\frac{1}{2}(8 - x) = \frac{1}{5}(x + 10) - x\)

Solve each problem. Write and solve an equation to answer each question. State what your variable represents.

33. Write and solve equations for each word sentence.
   A. Five more than 3 times a number is 38.
   B. Seven less than one-third of a number is 5.
   C. The product of 5 and a number is 18 more than twice the number.

34. At the end of a year, a bank account balance is $1319.17. The account had earned 6% simple interest during the year, and there had been no deposits or withdrawals. What was the balance at the beginning of the year?

35. Tia paid $46.28 for a sweater. That amount included a 4% sales tax. What was the price of the sweater?
**36.** **Challenge** In one week, Ed spent $12.25 more on entertainment than on all other items combined. During that week, he spent a total of $34.75. How much did Ed spend on entertainment?

**37.** **Challenge** There are four employees on a building crew: a laborer, two carpenters who each earn 50% more than the laborer, and a supervisor who earns 80% more than the laborer. The total payroll for the crew one week was $4205. How much did each employee earn that week?

**38.** **Challenge** The rectangle and triangle have the same perimeter. What is the value of \( n \)?
Error Analysis

It is important to be able to identify errors in solving equations and in solving application problems.

Identifying Errors in Equation Solutions

Example 1 Identify the student’s error and write a correct solution.

<table>
<thead>
<tr>
<th>x = 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x − 8 = 20</td>
</tr>
<tr>
<td>2x = 12</td>
</tr>
<tr>
<td>x = 6</td>
</tr>
</tbody>
</table>

Solution The student’s error is subtracting 8 from 20 on the right side. The correct step is to add 8 to both sides.

Correct solution:

| 2x − 8 = 20 |
| 2x = 28 |
| x = 14 |

TIP Check your solutions. A check for Example 1A shows that x = 6 is incorrect:

\[
2x − 8 = 20 \\
2 \cdot 6 − 8 \neq 20 \\
12 − 8 \neq 20 \\
4 \neq 20
\]
**B**

\[ 6 + \frac{n}{5} = 26 \]
\[ \frac{n}{5} = 20 \]
\[ n = 4 \]

**Solution**  The student’s error is dividing 20 by 5 on the right side. The correct step is to multiply both sides by 5.

Correct solution:

\[ 6 + \frac{n}{5} = 26 \]
\[ \frac{n}{5} = 20 \]
\[ n = 100 \]

**C**

\[ -7x - 9 = -30 \]
\[ -7x = -21 \]
\[ x = 3 \]

**Solution**  The student made a sign error when dividing \(-21\) by \(-3\). The correct solution is \(x = 3\).

**D**

\[ 3(r - 5) = 13 \]
\[ 3r - 5 = 13 \]
\[ 3r = 18 \]
\[ r = 6 \]
Solution  The student’s error is forgetting to distribute 3 on the left side. The correct step is to distribute 3; that is, multiply both r and −5 by 3.

One correct solution:

<table>
<thead>
<tr>
<th>3(r − 5) = 13</th>
<th>2x + 1 = 5x − 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>3r − 15 = 13</td>
<td>7x + 1 = −13</td>
</tr>
<tr>
<td>3r = 28</td>
<td>7x = −14</td>
</tr>
<tr>
<td>r = \frac{28}{3}</td>
<td>x = −2</td>
</tr>
</tbody>
</table>

Another correct solution:

<table>
<thead>
<tr>
<th>3(r − 5) = 13</th>
<th>2x + 1 = 5x − 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>\frac{3(r − 5)}{3} = \frac{13}{3}</td>
<td>\frac{2x + 1}{5} = \frac{5x − 13}{5}</td>
</tr>
<tr>
<td>r − 5 = \frac{13}{3}</td>
<td>−3x + 1 = −13</td>
</tr>
<tr>
<td>r − 5 + 5 = \frac{13}{3} + 5</td>
<td>−3x = −14</td>
</tr>
<tr>
<td>r = \frac{28}{3} ■</td>
<td>x = \frac{14}{3} ■</td>
</tr>
</tbody>
</table>

Solution  The student’s error is combining like terms that are on different sides of the equation. Like terms must be on the same side of the equation to be combined. The correct step is to subtract either 2x or 5x from both sides of the equation.

One correct solution:

<table>
<thead>
<tr>
<th>2x + 1 = 5x − 13</th>
<th>2x + 1 = 5x − 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>2x + 1 − 2x = 5x − 13 − 2x</td>
<td>2x + 1 − 5x = 5x − 13 − 5x</td>
</tr>
<tr>
<td>1 = 3x − 13</td>
<td>−3x + 1 = −13</td>
</tr>
<tr>
<td>14 = 3x</td>
<td>−3x = −14</td>
</tr>
<tr>
<td>\frac{14}{3} = x</td>
<td>x = \frac{14}{3} ■</td>
</tr>
</tbody>
</table>
Identifying Errors in Solutions to Application Problems

In solving application problems, students sometimes solve equations correctly, but make errors of other types, especially not writing the correct equations in problem solving.

Example 2 Identify the student’s error in the incorrect solution of each application problem. Find the correct solution.

A Application: Perimeter The perimeter of a rectangle is 60 feet, and its length is 20 feet. Find the width of the rectangle.

Incorrect solution:

\[
\begin{align*}
20w &= 60 \\
   w &= 3 \\
&\text{The width of the rectangle is 3 feet.}
\end{align*}
\]

Solution The student’s error is using the area formula instead of the perimeter formula. The correct formula for the perimeter of a rectangle is \(P = 2l + 2w\).

Correct solution:

\[
\begin{align*}
P &= 2l + 2w \\
   60 &= 2 \cdot 20 + 2w \\
   60 &= 40 + 2w \\
   20 &= 2w \\
   10 &= w \\
&\text{The width of the rectangle is 10 feet.}
\end{align*}
\]

B Application: Number Problem Six less than 3 times a number is 33. What is the number?

Incorrect solution:

\[
\begin{align*}
6 - 3n &= 33 \\
   -3n &= 27 \\
   n &= -9 \\
&\text{The number is } -9.
\end{align*}
\]
Solution  The student’s error is incorrectly translating *Six less than three times a number*. The correct translation is $3n - 6$, not $6 - 3n$.

Correct solution:

\[
\begin{align*}
3n - 6 &= 33 \\
3n &= 39 \\
 n &= 13
\end{align*}
\]

The number is 13. ■

Application: Distance, Rate, and Time  Mr. Vance drove 190 miles to visit a friend. First, he drove for $1\frac{1}{2}$ hours at an average rate of 60 miles per hour. Then he drove the rest of the way at an average rate of 40 miles per hour. What was Mr. Vance’s total driving time?

Incorrect solution:

\[
\begin{align*}
60 \cdot 1\frac{1}{2} + 40t &= 190 \\
90 + 40t &= 190 \\
40t &= 100 \\
t &= 2\frac{1}{2}
\end{align*}
\]

Mr. Vance’s total driving time was $2\frac{1}{2}$ hours.

Solution  The student’s error is using the solution to the equation as the answer to the question. The variable $t$ represents the number of hours driving at 40 miles per hour. The correct total driving time in hours was: $1\frac{1}{2} + 2\frac{1}{2} = 4$.

Correct answer:

Mr. Vance’s total driving time was 4 hours. ■
Problem Set

Identify the student’s error and write a correct solution.

1. $4x - 10 = 14$
   $4x = 14$
   $x = 1$

2. $-(n - 5) = 6$
   $-n - 5 = 6$
   $-n = 11$
   $n = -11$

3. $6(x - 1) = -48$
   $6x - 6 = -48$
   $6x = -54$
   $x = -9$

4. $8 = -8(p - 3)$
   $8 = -8p + 24$
   $-16 = -8p$
   $-2 = p$

5. $-13 = 2a - 8$
   $-5 = 2a$
   $-10 = a$

6. $1 = 35 - 5(x + 3)$
   $1 = 35 - 5x + 15$
   $1 = 38 - 5x$
   $-37 = -5x$
   $\frac{37}{5} = x$

7. $5a + 5 = -10$
   $5a = -15$
   $a = -3$

8. $2 + \frac{5}{3} = 11$
   $\frac{5}{3} = 13$
   $s = 39$
9.
\[
\begin{align*}
2 - \frac{x}{7} &= -2 \\
-\frac{x}{7} &= -4 \\
x &= -28
\end{align*}
\]

10.
\[
\begin{align*}
7 - 2x &= 6x + 14 \\
7 &= 4x + 14 \\
-7 &= 4x \\
-\frac{7}{4} &= x
\end{align*}
\]

11.
\[
\begin{align*}
3 - \frac{x + 4}{9} &= -2 \\
-\frac{x + 4}{9} &= -5 \\
-x + 4 &= -45 \\
x &= -49
\end{align*}
\]

12.
\[
\begin{align*}
7 - 2(5s - 4) + s &= 6s + 14 \\
7 - 10s + 8 + s &= 6s + 14 \\
-9s + 15 &= 6s + 14 \\
-3s + 15 &= 14 \\
-3s &= -1 \\
s &= \frac{1}{3}
\end{align*}
\]

13.
\[
\begin{align*}
-12a - 1 &= -49 \\
-12a &= -48 \\
a &= -4
\end{align*}
\]

14.
\[
\begin{align*}
3 + \frac{x}{3} &= 18 \\
\frac{x}{3} &= 15 \\
x &= 5
\end{align*}
\]

15.
\[
\begin{align*}
7d - 13 &= 5d + 11 \\
12d - 13 &= 11 \\
d &= 2
\end{align*}
\]

16.
\[
\begin{align*}
-2 &= 5x - 4(x + 7) \\
-2 &= 5x - 4x - 28 \\
-2 &= x - 28 \\
-30 &= x
\end{align*}
\]

ERROR ANALYSIS 179
Identify the student’s error in the incorrect solution of each application problem. Find the correct solution.

19. The area of a rectangle is 100 square feet, and its length is 40 feet. Find the width of the rectangle.
   Incorrect solution:
   \[
   A = 2l + 2w \\
   100 = 2 \cdot 40 + 2w \\
   100 = 80 + 2w \\
   20 = 2w \\
   10 = w \\
   \text{The width of the rectangle is} \\
   10 \text{ feet}
   \]

20. The product of 4 and a number is 12 more than 92. What is the number?
   Incorrect solution:
   \[
   4n + 12 = 92 \\
   4n = 80 \\
   n = 20 \\
   \text{The number is} 20.
   \]
21. The sum of three consecutive integers is 90. What is the greatest of the three integers?
   Incorrect solution:
   \[ x + (x + 1) + (x + 2) = 90 \]
   \[ x + x + 1 + x + 2 = 90 \]
   \[ 3x + 3 = 90 \]
   \[ 3x = 87 \]
   \[ x = 29 \]
   The greatest of the three integers is 29.

22. Margo and Nicole went shopping. Margo spent 50% more than Nicole. Together they spent a total of $90. How much did Nicole spend?
   Incorrect solution:
   Let \( x \) represent the amount Nicole spent.
   \[ x + 0.50x = 90 \]
   \[ 1.00x + 0.50x = 90 \]
   \[ 1.50x = 90 \]
   \[ x = \frac{90}{1.50} \]
   \[ x = 60 \]
   Nicole spent $60.
Inequalities

Working with inequalities is similar to working with equations.

Identifying Solutions of Inequalities

**DEFINITIONS**

An *inequality* is a mathematical sentence that compares numbers or expressions using one of the symbols $<$, $>$, $\leq$, or $\geq$.

A *solution* of a one-variable inequality is a value for the variable that makes the inequality true.

The set of all solutions of an inequality is the *solution set* of the inequality.

**Example 1** Identify the solution set of the inequality $x > -3$, using the replacement set $\{-5, -3.5, -3, -2, 0, 1\}$.

**Solution** Determine which of the numbers are to the right of $-3$ on a number line.

```
-6 -5 -4 -3 -2 -1 0 1 2
```

Using the replacement set, only the numbers $-2$, $0$, and $1$ are to the right of $-3$. The solution set is $\{-2, 0, 1\}$.

**Graphing Simple Inequalities**

Most inequalities that have a solution have an infinite number of solutions.

The *graph of a one-variable inequality* is the set of points on a number line that represents all the solutions of the inequality.

When graphing a one-variable inequality, draw an open dot if the endpoint is not a solution, and draw a solid dot if the endpoint is a solution. Draw a shaded arrow on one side of the endpoint to show the solutions. The following table shows some simple one-variable inequalities and their graphs.
### INEQUALITIES

**Solving and Graphing Inequalities**

Equivalent inequalities are inequalities that have the same solution set. The properties of order tell you how to obtain equivalent inequalities by adding, subtracting, multiplying, and dividing.

### Properties of Order

**Addition and Subtraction Properties of Order**

Adding or subtracting the same number on both sides of an inequality produces an equivalent inequality.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbols</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $a &gt; b$, then $a + c &gt; b + c$.</td>
<td>If $5 &gt; 2$, then $5 + 6 &gt; 2 + 6$.</td>
<td></td>
</tr>
<tr>
<td>If $a &lt; b$, then $a + c &lt; b + c$.</td>
<td>If $x &gt; 3$, then $x + 4 &gt; 3 + 4$.</td>
<td></td>
</tr>
<tr>
<td>If $a ≥ b$, then $a + c ≥ b + c$.</td>
<td>If $u + 5 ≤ 11$, then $u + 5 - 5 ≤ 11 - 5$.</td>
<td></td>
</tr>
<tr>
<td>If $a &gt; b$, then $a - c &gt; b - c$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If $a &lt; b$, then $a - c &lt; b - c$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If $a ≥ b$, then $a - c ≥ b - c$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If $a ≤ b$, then $a - c ≤ b - c$.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Multiplication and Division Properties of Order**

Multiplying or dividing both sides of an inequality by a positive number produces an equivalent inequality.

<table>
<thead>
<tr>
<th>For any $c &gt; 0$,</th>
</tr>
</thead>
<tbody>
<tr>
<td>If $a &gt; b$, then $a · c &gt; b · c$.</td>
</tr>
<tr>
<td>If $a &lt; b$, then $a · c &lt; b · c$.</td>
</tr>
<tr>
<td>If $a ≥ b$, then $a · c ≥ b · c$.</td>
</tr>
<tr>
<td>If $a ≤ b$, then $a · c ≤ b · c$.</td>
</tr>
</tbody>
</table>

| If $2 < 3$, then $4 · 2 < 4 · 3$. |
| If $2x ≥ 6$, then $\frac{2x}{2} ≥ \frac{6}{2}$. |
## Properties of Order (continued)

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbols</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplication and Division Properties of Order</td>
<td>For any ( c &lt; 0 ), if ( a &gt; b ), then ( a \cdot c &lt; b \cdot c ). If ( a &lt; b ), then ( a \cdot c &gt; b \cdot c ). If ( a \geq b ), then ( a \cdot c \leq b \cdot c ). If ( a \leq b ), then ( a \cdot c \geq b \cdot c ). If ( a &gt; b ), then ( \frac{a}{c} &lt; \frac{b}{c} ). If ( a &lt; b ), then ( \frac{a}{c} &gt; \frac{b}{c} ). If ( a \geq b ), then ( \frac{a}{c} \leq \frac{b}{c} ). If ( a \leq b ), then ( \frac{a}{c} \geq \frac{b}{c} ).</td>
<td>If ( \frac{a}{-3} &lt; 5 ), then ( -3(\frac{a}{-3}) &gt; -3 \cdot 5 ). If ( 12 &gt; 8 ), then ( \frac{12}{-4} &lt; -4 ).</td>
</tr>
</tbody>
</table>

Multiply both sides by \(-2\) and see what happens.

\[
5 > 2
\]
\[
5 \cdot (-2) \nless 2 \cdot (-2)
\]
\[
-10 < -4
\]

The method of solving an inequality is similar to the method of solving an equation. Use inverse operations along with the properties of order to obtain simpler inequalities that are equivalent. When you have the simplest equivalent inequality, you have the statement that best describes the solution set. When you have that statement, graph it.

**Example 2** Solve and graph each inequality.

**A** \( 5x - 10 < 60 \)

**Solution**

\[
5x - 10 < 60
\]
\[
5x - 10 + 10 < 60 + 10 \quad \text{Add 10 to both sides.}
\]
\[
5x < 70
\]
\[
\frac{5x}{5} < \frac{70}{5} \quad \text{Divide both sides by 5.}
\]
\[
x < 14
\]
B  \[-\frac{t}{3} \geq 2\]

Solution
\[-\frac{t}{3} \geq 2\]

\[-3 \cdot \left( -\frac{t}{3} \right) \leq -3 \cdot 2\] Multiply both sides by \(-3\). Reverse the inequality symbol.

\[t \leq -6\]

C  \[2(a + 2) - a > 0\]

Solution
\[2(a + 2) - a > 0\]

\[2a + 4 - a > 0\] Apply the distributive property.

\[a + 4 > 0\] Combine like terms.

\[a + 4 - 4 > 0 - 4\] Subtract 4 from both sides.

\[a > -4\]

D  \[3 > -\frac{5x + 1}{2}\]

Solution
\[3 > -\frac{5x + 1}{2}\]

\[3 > -\frac{1}{2}(5x + 1)\] Write \(-\frac{5x + 1}{2}\) as \(-\frac{1}{2}(5x + 1)\).

\[-2 \cdot 3 < -2 \cdot \left( -\frac{1}{2} \right)(5x + 1)\] Multiply both sides by \(-2\). Reverse the inequality symbol.

\[-6 < 5x + 1\]

\[-6 - 1 < 5x + 1 - 1\] Subtract 1 from both sides.

\[-7 < 5x\]

\[-\frac{7}{5} < \frac{5x}{5}\] Divide both sides by 5.

\[-\frac{7}{5} < x\]

REMEMBER
When graphing, draw an open dot for the symbols < and >. Draw a solid dot for the symbols \(\leq\) and \(\geq\).

REMEMBER
In general, \(a < b\) is equivalent to \(b > a\). In Example 2D, \(-\frac{7}{5} < x\) is equivalent to \(x > -\frac{7}{5}\).
Application: Comparing Membership Fees

Example 3  Eli is deciding whether to join gym A or gym B. Gym A costs $50 to join and $31.50 per month. Gym B costs $170 to join and $24 per month. For what length of time will it cost less to belong to gym A?

Solution  Let $x$ represent the number of months Eli belongs to a gym.

Cost of gym A < Cost of gym B

\[ 50 + 31.50x < 170 + 24x \]

Write an inequality to represent the situation.

\[ 50 + 31.50x - 24x < 170 + 24x - 24x \]

Subtract 24x from both sides so that the variable appears on only one side.

\[ 50 + 7.50x < 170 \]

\[ 7.50x < 120 \]

\[ \frac{7.50x}{7.50} < \frac{120}{7.50} \]

Divide both sides by 7.50.

\[ x < 16 \]

It will cost less to belong to gym A for any number of months less than 16 months. ■

Problem Set

Identify the solution set of the inequality, using the given replacement set.

1.  \( x > -5; \{-8, -5.01, -5, -4.8, 0, 4\} \)
2.  \( x < -1; \{-2, -1.6, -1, 0, 0.5\} \)
3.  \( x \geq 4; \{-4, -3, 0, 3.8, 4, 4.2, 5\} \)
4.  \( x \leq -3.5; \{-3.55, -3.5, -3.4, -3, 0, 1, 5\} \)
5.  \( x > -1; \left\{-4, -1.5, -\frac{1}{4}, -\frac{3}{5}, 0, \frac{1}{10}\right\} \)
6.  **Challenge** \( x < -3.2; \left\{-4, -\frac{3}{4}, -3.21, -3\frac{3}{20}, -3\frac{1}{8}, -3, 0\right\} \)
Solve and graph each inequality.

7. \(4x - 15 < 17\)

8. \(\frac{b}{2} \leq 4\)

9. \(-8 < 2x + 4\)

10. \(7x + 9 \geq 2\)

11. \(-\frac{v}{6} \geq -2\)

12. \(-5x - 25 > 30\)

13. \(1 - x \leq -6\)

14. \(6 - 3r - 25 > r + 1\)

15. \(\frac{x}{4} - 5 < -1\)

16. \(1 - \frac{t}{2} < -1\)

17. \(3(d + 5) > 30\)

18. \(2(x + 3) - 1 > 3\)

19. \(1 < \frac{4k + 3}{2}\)

20. \(-\frac{x + 3}{2} \leq -9\)

21. \(1 < \frac{-x - 1}{5}\)

22. \(x - 3(x + 1) - 7 > -x\)

23. **Challenge** \(4(a - 4) - 5(2a + 3) > 1 + a\)

24. **Challenge** \(\frac{1}{2}(w + 6) - 1 < \frac{1}{3}w\)

25. **Challenge** \(-10 < 3 + \frac{2x - 7}{2}\)

26. **Challenge** \(r - 5 > 3 - \frac{4r - 1}{3}\)

Solve each problem. Write and solve an inequality to answer each question. State what your variable represents.

27. Carlos wants to send a gift basket to his grandmother. Deliver Quick charges \$4.15 plus \$0.85 per pound. Ship Fast charges \$2.75 plus \$1.05 per pound.
   - **A.** For what weights will it cost less to use Ship Fast than Deliver Quick?
   - **B.** For what weight do the two companies charge the same amount? What is that charge?

28. Kathleen is a sales associate in a jewelry store. She earns \$560 per week plus an 8% commission on sales. How much does she need to sell in a week to earn at least \$700 that week?

29. The cost to ride a Sedan Service taxicab is \$2.90 plus \$1.80 per mile. The cost to ride a Green taxicab is \$2.15 plus \$1.95 per mile.
   - **A.** For what distances does it cost less to ride a Green taxicab than a Sedan Service taxicab?
   - **B.** For what distance is the cost the same? What is that cost?

30. **Challenge** The Golden Rectangle has been used in art and architecture since ancient times. It is a rectangle whose length is approximately 1.618 times its width. An artist has 1000 centimeters of framing material. He wants to frame a painting in the shape of a Golden Rectangle. Describe the dimensions he can use for the painting. Round dimensions to the nearest centimeter.

\[
\text{w} \\
\hline
\text{l} = 1.618w
\]

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