UNIT 6  Linear Equations and Inequalities

The yellow lines are parallel.
You’ve probably heard the phrase, “That’s where I draw the line!” In algebra, you can take this expression literally. Linear equations and their graphs play an important role in the never-ending quest to model the real world.

**Big Ideas**

► Expressions, equations, and inequalities express relationships between different entities.

► The laws of arithmetic can be used to simplify algebraic expressions and equations. Solving an equation means finding values for the variable or variables that make the equation a true statement.

► If you can create a mathematical model for a situation, you can use the model to solve other problems that you might not be able to solve otherwise. Algebraic equations can capture key relationships among quantities in the world.

**Unit Topics**

► Equations in Two Variables
► Graphs
► Lines and Intercepts
► Slope
► Slope-Intercept Form
► Point-Slope Form

► Parallel and Perpendicular Lines
► Equations From Graphs
► Applications: Linear Models
► Graphing Linear Inequalities
► Inequalities From Graphs


## Equations in Two Variables

An equation in the form $Ax + By = C$ is a linear equation in two variables.

### Checking Solutions to Equations in Two Variables

A solution to a linear equation in two variables is an ordered pair $(x, y)$. You can determine if an ordered pair is a solution to an equation in two variables by substituting the values of $x$ and $y$ into the equation. If a true statement results, the ordered pair is a solution to the equation.

**Example 1** Determine if each ordered pair is a solution to the equation $4x - y = 8$.

A. $(0, 8)$

**Solution** Substitute 0 for $x$ and 8 for $y$ in the equation $4x - y = 8$.

\[
4x - y = 8 \\
4 \cdot 0 - 8 = 8 \\
-8 \neq 8
\]

The ordered pair $(0, 8)$ is not a solution to the equation $4x - y = 8$. ■

B. $(−1, −12)$

**Solution** Substitute $−1$ for $x$ and $−12$ for $y$ in the equation $4x - y = 8$.

\[
4x - y = 8 \\
4 \cdot (-1) - (-12) = 8 \\
-4 + 12 = 8 \\
8 = 8 \checkmark
\]

The ordered pair $(−1, −12)$ is a solution to the equation $4x - y = 8$. ■

### Finding Solutions to an Equation in Two Variables

To find solutions to an equation in two variables, first solve the equation for $y$ in terms of $x$. Once you have the equation solved for $y$, choose some values of $x$ and substitute them to find values of $y$.

**Example 2** Find four solutions of the equation $6x + 3y = 9$.

(continued)
Solution

Step 1  Solve the equation for \( y \).

\[
6x + 3y = 9
\]

\[
6x + 3y - 6x = 9 - 6x \quad \text{Subtract} \ 6x \ \text{from each side.}
\]

\[
3y = 9 - 6x \quad \text{Simplify.}
\]

\[
\frac{3y}{3} = \frac{9 - 6x}{3} \quad \text{Divide each side by} \ 3.
\]

\[
y = \frac{9}{3} - \frac{6x}{3} \quad \text{Simplify.}
\]

\[
y = 3 - 2x \quad \text{Simplify.}
\]

Step 2  Using the values for \( x \) in the table below, find each value of \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -2 )</th>
<th>0</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( 7 )</td>
<td>( 3 )</td>
<td>( -5 )</td>
<td>( -13 )</td>
</tr>
</tbody>
</table>

When \( x = -2 \): \( y = 3 - 2 \cdot (-2) = 3 - (-4) = 3 + 4 = 7 \)

When \( x = 0 \): \( y = 3 - 2 \cdot 0 = 3 - 0 = 3 \)

When \( x = 4 \): \( y = 3 - 2 \cdot 4 = 3 - 8 = -5 \)

When \( x = 8 \): \( y = 3 - 2 \cdot 8 = 3 - 16 = -13 \)

The table below shows the values for \( y \).

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<table>
<thead>
<tr>
<th>( x )</th>
<th>( -2 )</th>
<th>0</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( 7 )</td>
<td>( 3 )</td>
<td>( -5 )</td>
<td>( -13 )</td>
</tr>
</tbody>
</table>

Four solutions are \((-2, 7), (0, 3), (4, -5), \) and \((8, -13)\). There are infinitely many other solutions to this equation.

Application: Admission Prices

You can use equations in two variables to solve real-world applications.

Example 3  At an art museum, admission tickets cost $12 for adults and $8 for children. On Sunday, 65 child tickets were sold and a total of $1780 was collected for admission. How many adult tickets were sold?

Solution

Step 1  Identify  Find the number of adult tickets sold.

Step 2  Strategize  Let \( T \) represent the total amount collected, \( a \) represent the number of adult tickets sold, and \( c \) represent the number of child tickets sold.

Step 3  Set Up  Write an equation to model the problem.
The problem is modeled by the equation $T = 12a + 8c$.

**Step 4** Solve Substitute 1780 into the equation for $T$ and 65 into the equation for $c$.

$T = 12a + 8c$

$1780 = 12a + 8 \cdot 65$

Solve the equation for $a$.

$1780 = 12a + 8 \cdot 65$ Write the equation.

$1780 = 12a + 520$ Multiply.

$1260 = 12a$ Subtract 520 from each side.

$105 = a$ Divide each side by 12.

105 adult tickets were sold.

**Step 5** Check

$T = 12a + 8c$ Write the original equation.

$1780 = 12 \cdot 105 + 8 \cdot 65$ Substitute 1780 for $T$, 105 for $a$, and 65 for $c$.

$1780 = 1260 + 520$ Multiply.

$1780 = 1780 \checkmark$ Add. ■

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**Problem Set**

Determine if the ordered pair is a solution to the equation $3x + 2y = 6$.

1. $(10, -12)$
2. $(8, -5)$
3. $(-2, 0)$
4. $(4, -3)$

Determine if the ordered pair is a solution to the equation $3x - 3y = 21$.

5. $(12, -5)$
6. $(5, -2)$
7. $(-1, 3)$
8. $(-3, -10)$

Determine if the ordered pair is a solution to the equation $2x - \frac{y}{2} = -8$.

9. $(3, 28)$
10. $(-5, -36)$
Solve the equation for \( y \) and then find the value of \( y \) for the given \( x \)-value.

11. \( 5x + y = 10 \)
   A. \( x = -2 \)
   B. \( x = 3 \)

12. \( 2x + 5y = 5 \)
   A. \( x = 15 \)
   B. \( x = -5 \)

13. \( 3y - 8x = 2 \)
   A. \( x = 2 \)
   B. \( x = -1 \)
   C. \( x = \frac{1}{2} \)

14. \( 3y - 6x = -3 \)
   A. \( x = -4 \)
   B. \( x = 2 \)

15. \( 2(x - 3) + 5 = 4y - 3(x + y) \)
   A. \( x = -2 \)
   B. \( x = 3 \)

16. \( 3x + 4y = 12 \)
   A. \( x = 8 \)
   B. \( x = -4 \)

17. \( 5x + 4y = 8 \)
   A. \( x = -4 \)
   B. \( x = 0 \)
   C. \( x = -3 \)

18. \( 4x - 2y = 10 \)
   A. \( x = -2 \)
   B. \( x = 5 \)

19. \( 2(2x - y) + 2 = y - 4 \)
   A. \( x = -3 \)
   B. \( x = 9 \)

20. \( 3x - 2y = -7 \)
   A. \( x = 3 \)
   B. \( x = -1 \)

21. \( 2(x + y) - 3(2x - 4) = 4(y - 2) \)
   A. \( x = 3 \)
   B. \( x = -1 \)
   C. \( x = -\frac{1}{3} \)

*22. Challenge \( \frac{1}{3}x - \frac{2}{3}y = 4 \)
   A. \( x = 12 \)
   B. \( x = -6 \)
   C. \( x = 3 \)

*23. Challenge \( \frac{x}{2} + \frac{2}{3}y = x - 2 \)
   A. \( x = -8 \)
   B. \( x = 6 \)

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Solve using an equation with two variables. For each problem:

A. Define variables for the unknowns.
B. Write an equation to model the problem.
C. Solve the equation.
D. Give your answer in a complete sentence.

24. An amusement park charges $21 for adults and $17 for children ages 12 and under. The Anderson family reunion included a trip to the amusement park, which cost a total of $706. If 8 adults and 7 teens were included in the trip, how many children ages 12 and under were included?

25. A wholesaler is packing books for shipping. A certain box will contain 24 novels, weighing 28 ounces each, and reference books weighing 52 ounces each. The total weight of the books is 984 ounces. How many reference books will be packed?
26. Lina bought 5 pounds of grapes at $2.59 per pound. She also bought peanuts at $1.95 per pound, and the total cost for peanuts and grapes was $26.60. How many pounds of peanuts did she buy?

27. In one week, a shirt shop sold 68 embroidered shirts and 97 silk-screened shirts, for a total of $2767.75 in sales. If the embroidered shirts sell for $17.95 each, what is the price of each silk-screened shirt?

28. Tickets to the junior class charity fundraising dance cost $15 for juniors and $18 for other students. Eighty juniors bought tickets, and total ticket sales were $2532. How many other students bought tickets?

29. Jake and Suki made a collage of photos for the yearbook. They used 30 large photos, each measuring 1.5 by 2 inches, and a number of small photos, each measuring 1.2 by 1.5 inches. The total area of the photos was 162 square inches. How many small photos did they use?

*30. Challenge* A designer has pleated 6 yards of fabric. Each large pleat decreases the total length of the fabric by 2.5 inches. Each small pleat decreases the length by 1.75 inches. After pleating, the fabric is 92.25 inches long. If the designer made 25 large pleats, how many small pleats did he make?
You have seen that you can use a number line to graph numbers. To graph an ordered pair, you need two number lines that form a coordinate plane.

**Definitions**

An ordered pair is a pair of numbers in which the first number is the $x$-coordinate, or abscissa, and the second number is the $y$-coordinate, or ordinate, of a point's location.

The Cartesian coordinate system is a method of locating points in a plane in which the coordinates of the points are its distances from two intersecting perpendicular lines called axes. The horizontal line, often called the $x$-axis, and the vertical line, often called the $y$-axis, intersect at the origin. The ordered pair at the origin is (0, 0) and is labeled $O$.

**Graphing on a Coordinate Plane**

**How to Graph a Point on a Coordinate Plane**

1. Start at the origin.
2. Move left or right along the $x$-axis according to the $x$-coordinate of the ordered pair. Move right for positive numbers and left for negative numbers.
3. Move up or down according to the $y$-coordinate. Move up for positive numbers and down for negative numbers.

**Example 1** Graph the points $A(1, 5)$, $B(0, -3)$, and $C(-2, 4)$ on the coordinate plane.

**Solution**

To graph $A(1, 5)$, start at the origin and move 1 unit to the right along the $x$-axis and 5 units up parallel to the $y$-axis.

To graph $B(0, -3)$, start at the origin. Since the $x$-coordinate is 0, you do not move any distance along the $x$-axis. Move 3 units down along the $y$-axis.

To graph $C(-2, 4)$, start at the origin and move 2 units to the left and 4 units up.
Identifying Points on a Coordinate Plane

Example 2  Identify the coordinates of points $D$, $E$, and $F$.

Solution
Point $D$ is 4 units to the left of the origin and 4 units up. The coordinates are $(-4, 4)$.
Point $E$ is 2 units to the right of the origin and 0 units up. The coordinates are $(2, 0)$.
Point $F$ is 4.5 units to the right of the origin and 2 units down. The coordinates are $(4.5, -2)$.

Identifying Quadrants

The coordinate plane is divided into four regions called quadrants. The quadrants are labeled using the Roman numerals I, II, III, and IV. Points in Quadrant I have a positive $x$-coordinate and a positive $y$-coordinate. In Quadrant II, points have a negative $x$-coordinate and a positive $y$-coordinate. Points in Quadrant III have a negative $x$-coordinate and a negative $y$-coordinate. In Quadrant IV, points have a positive $x$-coordinate and a negative $y$-coordinate. Points that lie on one of the axes are not in any quadrant.

Example 3  Identify the quadrant in which each point lies.
A. $Q(-2, 8)$
Solution  The $x$-coordinate is negative and the $y$-coordinate is positive, so the point lies in Quadrant II.
B. $Z(0, 2)$
Solution  The $x$-coordinate is zero and the $y$-coordinate is positive, so the point lies on the $y$-axis.
C. $B(4, -3)$
Solution  The $x$-coordinate is positive and the $y$-coordinate is negative, so the point lies in Quadrant IV.
D. $(4, 3)$
Solution  The $x$-coordinate is positive and the $y$-coordinate is positive, so the point lies in Quadrant I.

Application: City Map

Example 4  A city planner uses the coordinate plane to identify locations in the city. Use the map to identify the coordinates of each of the following locations.
A. The Post Office
B. Center City Park
C. Patterson High School
D. The City Courthouse

(continued)
**Solution**

A. The Post Office is located 9 blocks west of Main Street and 3 blocks south of Center Street. Its coordinates are \((-9, -3)\).

B. Center City Park is on Center Street, 3 blocks west of Main Street. Its coordinates are \((-3, 0)\).

C. Patterson High School is located 9 blocks east of Main Street and 9 blocks south of Center Street. Its coordinates are \((9, -9)\).

D. The City Courthouse is located 7 blocks east of Main Street and 8 blocks north of Center Street. Its coordinates are \((7, 8)\).

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**Problem Set**

Graph the points on the coordinate plane.

1. \(A(3, -2), B(0, 5), C(-2, -1)\)
2. \(D(4, 1), E(3, -4), F(-2, 3)\)
3. \(R(-3, 0), S(0, -3), T(2, 5), U(5, 2)\)
4. \(K(-3, 1), L(0, 3), M(2, -2), N(-2, -4)\)
5. \(F(-2, -5), G(1, 2.5), H(1, -2), J(-3.5, 0)\)
6. \(A(2, -2.5), B(-1, 4), C(0.5, -3), D(-4, -4)\)
7. \(Q(1, 1), R(1, -1), S(-6, -4), T(-3, 2), U(-6, 0), V(8, 9)\)

*8. Challenge* \(A(-8, -7), B(-6, 1.5), C(2.5, 9.5), D(7, 3), E(4, -3.5), F(0, -6.5)\)

Identify the coordinates for each point.

9. [Graph of points A, B, C, D, E, F with coordinates]
10. [Graph of points P, Q, R, S with coordinates]
11. [Graph of points M, N, O, P, Q with coordinates]
12. [Graph of points W, X, Y, Z with coordinates]
Identify the quadrant in which each point with the given coordinates lies. If the point lies on an axis, name the axis.

17. \((-3, -1)\)  
18. \((-5, 0)\)  
19. \((3, 4, 9)\)  
20. \((6, -4)\)

Give the coordinates and quadrant for each point.

22. A. Point \(P\)  
B. Point \(M\)
23. A. Point \(H\)  
B. Point \(J\)
24. A. Point \(T\)  
B. Point \(Z\)
25. A. Point \(L\)  
B. Point \(N\)
Use the maps to answer the questions.

26.  

A. Name the plants that appear at each set of coordinates on the garden map: (−3, 3), (3, 1), (1, −4).
B. Give the coordinates for the gate and the scarecrow.

27.  

Create a map of a town with the following locations:
A. Main Street on the x-axis and Elm Street on the y-axis.
B. City Hall at (−4, −6), a High School at (8, 9), and a Supermarket at (−6, 1).
C. A rectangular park whose northwest corner is located 6 squares west and 2 squares south of the High School, and whose southeast corner is located 9 squares east and 8 squares north of the City Hall.

28.  

Kevin made this map of the trees around his house.
A. Give the coordinates for the oak tree.
B. Give the coordinates for the 3 maple trees.

*29.  Challenge  

A. Use the map of Mountainview Nature Center to give coordinates for the following features: Picnic Area, Scenic Vista, Marina, Restroom, Waterfall, Boulder Island, and the southwest corner of the Visitors’ Center building.
B. Tell where you would be at each of these points on the map: (9, −6), (−8, −7), (4, 4), (−2.5, −2.5), (−5, 7).

*30. Challenge  

Jacinta made this map of the approximate locations of stars she saw in the winter sky.
A. Give the coordinates for Castor, Procyon, Sirius, and Betelgeuse.
B. Name the star that is closest to each set of coordinates: (−3.5, 3), (0.5, 4.5), (1.7, −4.3), (2.3, 1).
The graph of a linear equation is a line.

**Definition**

The standard form of a linear equation is $Ax + By = C$, where $A$, $B$, and $C$ are integers and $A$ and $B$ are not both zero.

**Writing Equations in Standard Form**

**Example 1** Write each equation in standard form.

A. $4x = y + 12$

Solution

Subtract $y$ from both sides.

$4x - y = 12$

The equation in standard form is $4x - y = 12$. ■

B. $y = 3x - 18$

Solution

Subtract $3x$ from both sides.

$y - 3x = 3x - 18 - 3x$

$-3x + y = -18$

The equation in standard form is $-3x + y = -18$. ■

**Graphing Lines on the Coordinate Plane**

You can use ordered pairs to graph a line on the coordinate plane. If you graph any two points on the plane, there is exactly one line that passes through both points. So, to graph a line you need to know at least two points on the line.

**Example 2** Graph the line containing the points (0, 2), (1, 5), and (−2, −4).

Solution Graph each point. Then draw the line connecting the three points.

**Tip** Though two points determine a line, it is often helpful to graph three points as a way of checking your work.
Collinear Points

Points are **collinear** if they lie on the same line.

**Example 3**  Are the points (4, 0), (6, 2), and (−1, −1) collinear?

**Solution**  Graph each point. Then try to draw a line connecting the three points.

![Graph](image)

A line cannot be drawn connecting the three points. The points are not collinear. ■

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**To Graph Lines Using Intercepts**

When the equation of a line is in standard form, it is convenient to use the \(x\)- and \(y\)-intercepts to graph the line.

**Definition**

The \(x\)-intercept of a graph is the \(x\)-coordinate of the point where the graph intersects the \(x\)-axis. The \(y\)-intercept is the \(y\)-coordinate of the point where the graph intersects the \(y\)-axis.

Any point that lies on the \(x\)-axis has a \(y\)-coordinate of 0 and any point that lies on the \(y\)-axis has an \(x\)-coordinate of 0. So, to find the \(x\)-intercept of a line, find the value of \(x\) when \(y = 0\), and to find the \(y\)-intercept, find the value of \(y\) when \(x = 0\).
Example 4  Graph the line $5x - 4y = 20$ by finding its $x$- and $y$-intercepts.

Solution

Step 1  Find the $x$- and $y$-intercepts.

To find the $x$-intercept, substitute 0 into the equation for $y$. Then solve for $x$.

$5x - 4y = 20$  Write the equation of the line.

$5x - 4 \cdot 0 = 20$  Substitute 0 for $y$.

$5x - 0 = 20$

$5x = 20$  Solve for $x$.

$\frac{5x}{5} = \frac{20}{5}$

$x = 4$

The $x$-intercept is 4, so the line intersects the $x$-axis at the point $(4, 0)$.

To find the $y$-intercept, substitute 0 into the equation for $x$. Then solve for $y$.

$5x - 4y = 20$  Write the equation of the line.

$5 \cdot 0 - 4y = 20$  Substitute 0 for $x$.

$0 - 4y = 20$

$-4y = 20$

$-4y = 20$

$y = -5$

The $y$-intercept is $-5$, so the line intersects the $y$-axis at the point $(0, -5)$.

Step 2  Graph the $x$- and $y$-intercepts and connect the points to draw the line.

Application: Purchase

Example 5  You are buying food for a family reunion and have purchased 120 hot dogs. Hot dog rolls come in packages of 8 and packages of 10. The situation can be modeled by the linear equation

$8x + 10y = 120$

where $x$ is the number of packages containing 8 rolls and $y$ is the number of packages containing 10 rolls.

A.  Find the $x$- and $y$-intercepts and graph the equation $8x + 10y = 120$.

B.  What is the domain for this problem?

C.  Name all possible choices for the number of packages of each quantity of hot dog rolls you could buy.

(continued)
Solution

A. To find the $x$-intercept, substitute 0 into the equation for $y$. Then solve for $x$.

$$8x + 10y = 120$$
$$8x + 10 \cdot 0 = 120$$
$$8x = 120$$
$$x = 15$$

The $x$-intercept is 15.

To find the $y$-intercept, substitute 0 into the equation for $x$. Then solve for $y$.

$$8x + 10y = 120$$
$$8 \cdot 0 + 10y = 120$$
$$10y = 120$$
$$y = 12$$

The $y$-intercept is 12.

B. Since only a whole number of packages can be purchased, only the whole number solutions are meaningful. The domain is all whole numbers greater than or equal to 0 and less than or equal to 15.

C. Use a table to list all the whole number coordinates the line passes through.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>12</td>
<td>8</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

From the table, read the four possible choices: 0 packages of 8 rolls and 12 packages of 10 rolls, 5 packages of 8 rolls and 8 packages of 10 rolls, 10 packages of 8 rolls and 4 packages of 10 rolls, and 15 packages of 8 rolls and 0 packages of 10 rolls.
Problem Set

Write each equation in standard form.
1. \( x = 14 - 2y \)
2. \( 6y - 12 = -3x \)
3. \( \frac{x}{3} + 4 = \frac{3}{4}y \)
4. \( 10 - 2(x - y) = 3x + 1 \)
5. \( 4(x - 2) = 3(2y - 1) \)
*6. Challenge \( 3(x - y) - \frac{x}{2} - 4 = 5y - 2(2y - 3x) - 6 \)

Find the \( x \)- and \( y \)-intercepts.
7. \[ \text{Graph} \]
8. \( 3x - 2y = 18 \)
9. \[ \text{Graph} \]
10. \( -6x + 4y = 96 \)

Graph the points and determine if they are collinear.
11. \( (1, 0), (3, 4), (-2, -3) \)
12. \( (8, -5), (-6, 2), (0, -1) \)
13. \( (-3, 8), (2, 4), (9, -1) \)

Graph the line containing each set of points.
14. \( (-3, -3), (9, 7), (-9, -8) \)
15. \( (7, -2), (0, 4), (-7, 10) \)
16. \( (-7, -2), (7, 0), (-1, -1) \)
17. \( (-5, 4), (1, -4), (-8, 8) \)
18. \( (2, 8), (-1, 2), (-7, -10) \)
19. \( (5, 7.5), (9, 7), (-7, 9) \)

For each problem:
A. Find the \( x \)- and \( y \)-intercepts.
B. Use the intercepts to graph the line.
20. \( 3x + 2y = 12 \)
21. \( x - 3y = -9 \)
22. \( 4x - 3y = -24 \)
23. \( 2(x - 3y) + 2 = 2(5 - y) \)
24. \[ \text{Challenge} \]
25. \[ \text{Challenge} \]
26. A florist is making flower arrangements and each arrangement will include either 6 or 9 daisies. He is using 108 daisies in all. How many of each type of arrangement could he make?

27. Aleksandra is buying nuts for a large party. She has $112, and wants to buy cashews at $7 per pound and pistachios at $8 per pound. How many pounds of each kind of nut could she buy?

28. The senior class trip will include 180 students who will ride in taxis in groups of 5 or 6 students. How many of each size group could the students make?

29. Malachi is pouring punch into cups that hold 3 or 4 ounces. He has 72 ounces of punch in all; how many of each size cup could he fill?
Slope

The steepness of a line is its slope.

The slope of a line is the ratio of the vertical change, or rise, between any two points on the line to the horizontal change, or run, between the same two points.

\[
\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}}
\]

**SLOPE FORMULA**

The slope \( m \) of a line containing the two points \((x_1, y_1)\) and \((x_2, y_2)\) can be found using the formula

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

**THINK ABOUT IT**

\( y_2 - y_1 \) is the vertical change and \( x_2 - x_1 \) is the horizontal change.

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**Finding the Slope of a Line**

To find the slope of a line, you can use the graph of the line to find the rise and run or you can use the slope formula if you know two points that lie on the line.

**Example 1**

**A.** Find the slope of the line shown in the graph using the ratio of rise to run.

**Solution** The vertical change between the two points is 4 units. The horizontal change between the two points is 2 units.

\[
\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2
\]

The slope of the line is 2. ■

(continued)
B. Find the slope of the line passing through the points \((-3, 7)\) and \((1, 2)\).

**Solution**  Use the slope formula. Choose either point to be \((x_1, y_1)\):

\[
\begin{align*}
(1, 2) & \quad (-3, 7) \\
(x_1, y_1) & \quad (x_2, y_2)
\end{align*}
\]

Substitute values into the slope formula for \(x_1, x_2, y_1,\) and \(y_2\):

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 2}{-3 - 1} = \frac{5}{-4} = -\frac{5}{4}
\]

The slope of the line is \(-\frac{5}{4}\). ■

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**Finding the Slope of Horizontal and Vertical Lines**

**Example 2** Find the slope of each line.

**A.** The points \((0, 5)\) and \((4, 5)\) lie on the line. Use the slope formula.

\[
\begin{align*}
(0, 5) & \quad (4, 5) \\
(x_1, y_1) & \quad (x_2, y_2)
\end{align*}
\]

Substitute values into the slope formula for \(x_1, x_2, y_1,\) and \(y_2\):

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 5}{4 - 0} = \frac{0}{4} = 0
\]

The slope of the line is 0. All horizontal lines have a slope of 0. ■

**B.** The points \((7, 6)\) and \((7, -3)\) lie on the line. Use the slope formula.

\[
\begin{align*}
(7, 6) & \quad (7, -3) \\
(x_1, y_1) & \quad (x_2, y_2)
\end{align*}
\]

Substitute values into the slope formula for \(x_1, x_2, y_1,\) and \(y_2\):

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 6}{7 - 7} = \frac{-9}{0}
\]

Division by zero is undefined, so the slope of the line is undefined. All vertical lines have an undefined slope. ■
Using the Equation of a Line to Determine the Slope

Example 3  Find the slope of the line $3x + 5y = 30$.

Solution

Step 1  You need two points to find the slope of the line. Intercepts are easy points to find, so use them to compute the slope.

Find the $x$- and $y$-intercepts of the line.

To find the $x$-intercept, substitute 0 into the equation $y$. Then solve for $x$.

$3x + 5y = 30$
$3x + 5 \cdot 0 = 30$
$3x = 30$
$x = 10$

The $x$-intercept is 10.

To find the $y$-intercept, substitute 0 into the equation for $x$. Then solve for $y$.

$3x + 5y = 30$
$3 \cdot 0 + 5y = 30$
$5y = 30$
$y = 6$

The $y$-intercept is 6.

The points $(10, 0)$ and $(0, 6)$ lie on the line $3x + 5y = 30$.

Step 2  Use the intercepts to find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{10 - 10} = \frac{6}{-10} = \frac{-3}{5}$$

The slope of the line $3x + 5y = 30$ is $-\frac{3}{5}$. ■
Graphing a Line Using a Point on the Line and the Slope

Example 4  
Graph the line that contains the point (1, 6) and has a slope of $\frac{2}{3}$.

Solution  
The slope of the line is $\frac{2}{3}$, so the vertical change, $y_2 - y_1$, is 2 and the horizontal change, $x_2 - x_1$, is 3. Since you know (1, 6) is a point on the line, you can set up and solve the two equations:

\[
\begin{align*}
  y_2 - y_1 &= 2 \\
  x_2 - x_1 &= 3 \\
  y_2 - 6 &= 2 \\
  x_2 - 1 &= 3 \\
  y_2 - 6 + 6 &= 2 + 6 \\
  x_2 - 1 + 1 &= 3 + 1 \\
  y_2 &= 8 \\
  x_2 &= 4
\end{align*}
\]

A second point on the line is (4, 8). Now plot the two points, (1, 6) and (4, 8), and draw the line.

Application: Rate of Change

A rate of change is the ratio of a change in one quantity to a change in a second quantity. A linear equation models a constant rate of change.

Example 5  
The table shows the total late fee for a movie rental. Find the rate of change of late fee to time.

<table>
<thead>
<tr>
<th>Time (days late)</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Late Fee (dollars)</td>
<td>1.50</td>
<td>3</td>
<td>4.50</td>
</tr>
</tbody>
</table>

Solution  
Write the rate of change as a ratio. Start by picking two ordered pairs.

\[
\text{rate of change} = \frac{\text{change in late fee}}{\text{change in time}} = \frac{3 - 1.50}{4 - 2} = \frac{1.50}{2} = 0.75
\]

Choose two more ordered pairs to see that the rate of change is constant.

\[
\begin{align*}
  \text{rate of change} &= \frac{\text{change in late fee}}{\text{change in time}} \\
  &= \frac{4.50 - 3}{6 - 4} = \frac{1.50}{2} = 0.75
\end{align*}
\]

The rate of change is $0.75 per day. ■
Problem Set

Find the slope of the line using the ratio of rise to run.

1. Use the slope formula to find the slope of the line passing through the given points.
   4. (4, 8) and (1, 6)
   5. (0, 2) and (0, 9)
   6. (−4, 0) and (7, 0)

Determine if the slope of the line is positive, negative, zero, or undefined.

10. Use the slope formula to find the slope of the line passing through the given points.
   7. (−2, 9) and (−10, 33)
   8. (12, 5) and (18, 14)
   9. (−4, 7) and (0, 8)
For each problem:
A. Find two points that lie on the line.
B. Use the slope formula to find the slope of the line.

14. \[ 2 \mathrm{line} - 4y = 4 \]
15. \[ -3x + 5y = 30 \]
16. \[ 4x - 4y = 16 \]
17. \[ -3x + 5y = 30 \]
18. \[ -7x - 2y = 56 \]
19. \[ -12x + 3y = -72 \]
20. \[ 13x - 5y = 130 \]

Graph the line that contains the given point and has the given slope.

21. \( (3, 1); m = \frac{1}{3} \)
22. \( (-4, 7); m = 0 \)
23. \( (9, 1); \) slope is undefined
24. \( (-5, 4); m = -\frac{3}{4} \)

Solve. For each problem:
A. Find the rate of change between each set of points.
B. Use your answer from part A to determine if the relation is linear.
C. Explain why or why not.

25. The table shows the cost for movie tickets. Find the rate of change of price to number of tickets.

<table>
<thead>
<tr>
<th>Number of Tickets</th>
<th>1</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$6.50</td>
<td>$19.50</td>
<td>$32.50</td>
</tr>
</tbody>
</table>

26. The table shows the total number of miles that Juanita has driven this week to work. Find the rate of change of miles driven to time.

<table>
<thead>
<tr>
<th>Number of Days</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Miles</td>
<td>48</td>
<td>64</td>
<td>80</td>
</tr>
</tbody>
</table>

27. The table shows the total profit that Domingo’s Barber Shop has earned this month. Find the rate of change of profit earned to the number of haircuts.

<table>
<thead>
<tr>
<th>Haircuts</th>
<th>13</th>
<th>27</th>
<th>55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>$45.50</td>
<td>$94.50</td>
<td>$192.50</td>
</tr>
</tbody>
</table>

28. The table shows the cost for ride tickets for the local carnival. Find the rate of change of price to number of tickets.

<table>
<thead>
<tr>
<th>Number of Tickets</th>
<th>15</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$30</td>
<td>$54</td>
<td>$105</td>
</tr>
</tbody>
</table>

29. **Challenge** The table shows the total amount of minutes that Katarina has spent grocery shopping this year. Find the rate of change of minutes spent shopping to time.

<table>
<thead>
<tr>
<th>Month</th>
<th>June</th>
<th>September</th>
<th>November</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Minutes</td>
<td>480</td>
<td>720</td>
<td>880</td>
</tr>
</tbody>
</table>

30. **Challenge** The table shows the total amount of hours that Marta has studied for the upcoming College Entrance Exam. Find the rate of change of hours spent studying to time.

<table>
<thead>
<tr>
<th>Date</th>
<th>16th</th>
<th>17th</th>
<th>18th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Hours</td>
<td>12</td>
<td>18</td>
<td>24</td>
</tr>
</tbody>
</table>
You can write the equation of a line in various forms including standard form and slope-intercept form.

The different forms of the equation of a line are equivalent. That is, even though they are written differently, they represent the same line so they will have the same graph and the same solutions.

The different forms can help identify characteristics of a line more easily. For example, the slope-intercept form of a linear equation can help you identify the slope and y-intercept, while the slope may not be easily identifiable from the same equation written in standard form.

**DEFINITION**
The slope-intercept form of a linear equation is $y = mx + b$, where $m$ is the slope of the line and $b$ is the y-intercept.

**Identifying the Slope and y-Intercept**

**Example 1** Identify the slope and y-intercept of each line.

A. $y = 3x + 4$

**Solution**

A. The equation of the line is in slope-intercept form. $m$ is the slope of the line and $b$ is the y-intercept.

\[
\begin{align*}
&y = mx + b \\
&y = 3x + 4 \\
\text{The slope of the line is 3 and the y-intercept is 4.}
\end{align*}
\]

B. $y = -5x - \frac{1}{2}$

**Solution**

B. Write the equation as $y = -5x + \left(-\frac{1}{2}\right)$.

\[
\begin{align*}
&y = mx + b \\
&y = -5x + \left(-\frac{1}{2}\right) \\
\text{The slope of the line is -5 and the y-intercept is -}\frac{1}{2}.
\end{align*}
\]
Converting a Line from Standard to Slope-Intercept Form

Example 2  Write the equation of the line $2x + 5y = 15$ in slope-intercept form.

Solution  Solve the equation for $y$.

$2x + 5y = 15$

$5y = -2x + 15$  Subtract $2x$ from each side of the equation.

$y = -\frac{2}{5}x + 3$  Divide each side by 5.

The equation in slope-intercept form is $y = -\frac{2}{5}x + 3$. ■

Using Slope-Intercept Form to Graph a Line

Example 3  Graph the line $3x + y = 5$.

Solution

Step 1  Write the equation in slope-intercept form.

$3x + y = 5$

$y = -3x + 5$  Subtract $3x$ from each side of the equation.

Step 2  Use the equation to identify the slope and $y$-intercept of the line.

$y = mx + b$

$y = -3x + 5$

The slope of the line is $-3$ and the $y$-intercept is 5.

Step 3  Use the $y$-intercept to graph a point on the line. The line intersects the $y$-axis at the point $(0, 5)$.

Step 4  Use the slope of the line to graph a second point on the line. The slope of the line is $-3$ or $\frac{-3}{1}$. The rise is $-3$ and the run is 1. Start at the point $(0, 5)$. Move 3 units down and 1 unit to the right. A second point on the line is $(1, 2)$. Draw a line through the points $(0, 5)$ and $(1, 2)$.

A linear equation is in standard form when it is in the form: $Ax + By = C$. 
Application: Profit

Example 4  You are selling items on an online auction site. Each item sells for $2.50 and there is a one-time listing fee of $8.

A. Write an equation to model your profit, $P$.

B. Graph the equation.

C. What does each intercept represent?

D. What is the domain of the problem?

Solution

A. Profit = Cost of Each Item × Number of Items Sold − Fees

\[
P = 2.50 \times n - 8
\]

The problem is modeled by the equation $P = 2.5n - 8$.

B. Find the slope and y-intercept of the line.

\[
y = mx + b
\]

\[
P = 2.5n + (-8)
\]

The slope of the line is 2.5 and the y-intercept is −8. Use the slope of the line to graph a second point on the line. Write the slope of the line as $\frac{2.5}{1}$. The rise is 2.5 and the run is 1. Start at the point (0, −8).

Move 2.5 units up and 1 unit to the right. A second point on the line is (1, −5.5). Draw a line through the points (0, −8) and (1, −5.5).

C. The vertical axis represents $P$, the dollar amount of your profit. The $P$-intercept, (0, −8) represents the greatest amount of money you can lose, or the greatest negative profit amount. If you sell 0 items, you will still have to pay the listing fee of $8. Your profit will be −$8. The horizontal axis represents $n$, the number of items sold. The line crosses the $n$-axis between 3 and 4, at the point (3.2, 0). If you sold 3.2 items, you would break even. Any $n$-value greater than that number signifies a positive profit amount. Since you can sell only whole items, you must sell 4 items to make a profit.

D. Since you can only sell whole items, the domain of the problem is all whole numbers.
Problem Set

Identify the slope and $y$-intercept.

1. $y = 2x - 8$
2. $y = -\frac{1}{2}x + 7$
3. $y = \frac{1}{4}x - \frac{3}{4}$
4. $y = 5x + 12$
5. $y = -7x + \frac{7}{2}$
6. $y = 2x - 8$

Write each equation in slope-intercept form.

7. $4x - 7y = 14$
8. $3x + 4y = 24$
9. $9x - 2y = -36$
10. $8x + 4y = 48$
11. $-3x - 2y = 30$
12. $-6x - 9y = -36$

Graph the line.

13. $2x + y = 9$
14. $-4x + y = 5$
15. $-6x - y = 3$
16. $\frac{1}{2}x + y = 4$
17. $x + y = 7$
18. $x - 2y = 10$
19. $-3x + y = -2$
20. $x + 5y = 15$
21. $-x - y = 1$

Solve. For each problem:

A. Write an equation to model the problem.
B. Graph the equation.
C. Describe the practical meaning of the $y$-intercept.
D. Answer the question.

22. You are selling items at a fair for $6.50 each. Renting the booth at the fair cost $25. How many items do you need to sell to make a profit?

23. Manya is selling charm bracelets at the school flea market. Each bracelet sells for $3 and there is a charge of $0.25 per charm. How much does a bracelet cost with five charms?

24. Olayinka is participating in a bowl-a-thon to benefit the local hospital. Each sponsor will pay Olayinka $2 plus $0.10 per pin that she knocks down. How much will Olayinka raise per sponsor for knocking down 52 pins?

25. Zain is getting her pictures printed at the local photo shop. The cost to print digital pictures is a set $5.50 fee per order plus an additional $0.05 per print. How much will Zain pay to have 25 pictures printed?

26. Moving Video charges a one-time membership fee of $9.99 plus $3.99 to rent each movie. How much will renting 22 movies cost?

27. **Challenge** Carlos is selling dried fruit bars for the football team. Each bar sells for $0.50 and the cost of ordering the box of 100 bars was $12. How many candy bars must Carlos sell to make a profit of $30?

28. **Challenge** You are selling items on an online auction site. Each item sells for $3.25 and there is a one-time listing fee of $14. How many items must you sell to make a profit of $100?
You can write the equation of a line in point-slope form if you know the slope of the line and any point that lies on the line.

**DEFINITION**

The point-slope form of the equation of a line that passes through the point \((x_1, y_1)\) and has a slope of \(m\) is \(y - y_1 = m(x - x_1)\).

You can see how the point-slope form is derived using the slope formula.

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Write the slope formula.

\[
m(x_2 - x_1) = y_2 - y_1
\]

Multiply each side by \((x_2 - x_1)\).

\[
m(x_2 - x_1) = y_2 - y_1
\]

Simplify.

\[
y - y_1 = m(x - x_1)
\]

Symmetric Property

\[
y - y_1 = m(x - x_1)
\]

Substitute \(y\) for \(y_2\) and \(x\) for \(x_2\).

**Writing the Equation of a Line in Point-Slope Form**

**Example 1** Use point-slope form to write an equation of the line that passes through the given point and has the given slope.

**A.** \((3, 5);\) slope \(= -\frac{1}{2}\)

**Solution**

\[
y - y_1 = m(x - x_1)
\]

\[
y - 5 = -\frac{1}{2}(x - 3)
\]

The equation of the line in point-slope form is \(y - 5 = -\frac{1}{2}(x - 3)\). ■

**B.** \((-1, -6);\) slope \(= 5\)

**Solution**

\[
y - y_1 = m(x - x_1)
\]

\[
y + 6 = 5(x + 1)
\]

The equation of the line in point-slope form is \(y + 6 = 5(x + 1)\). ■
Using the Graph of a Line to Write the Equation in Point-Slope Form

Example 2

A. Write the equation of the line shown in the graph using the coordinates (5, 4).

B. Write the equation of the line shown in the graph using the coordinates (–9, –3).

C. Determine if the two equations are equivalent.

Solution

A.

Step 1 Find the slope of the line using the slope formula.
\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-3)}{5 - (-9)} = \frac{4 + 3}{5 + 9} = \frac{7}{14} = \frac{1}{2} \]

Step 2 Write the equation in point-slope form. Use point (5, 4) in the point-slope equation.

\[ y - y_1 = m(x - x_1) \]

\[ y - 4 = \frac{1}{2}(x - 5) \]

B.

Step 1 Use the slope found in part A: \( \frac{1}{2} \).

Step 2 Write the equation in point-slope form. Use the point (–9, –3).

\[ y - y_1 = m(x - x_1) \]

\[ y - (-3) = \frac{1}{2}[x - (-9)] \]

\[ y + 3 = \frac{1}{2}(x + 9) \]
C. Show that the equations in Step 2 are equivalent by writing each in slope-intercept form.

Solve the first equation for \( y \).
\[
y - 4 = \frac{1}{2} (x - 5)
\]
Solve the second equation for \( y \).
\[
y + 3 = \frac{1}{2} (x + 9)
\]
\[
y - 4 = \frac{1}{2} x - \frac{5}{2}
\]
\[
y + 3 = \frac{1}{2} x + \frac{9}{2}
\]
\[
y = \frac{1}{2} x - \frac{5}{2} + 4
\]
\[
y = \frac{1}{2} x + \frac{9}{2} - 3
\]
\[
y = \frac{1}{2} x - \frac{5}{2} + \frac{8}{2}
\]
\[
y = \frac{1}{2} x + \frac{9}{2} - \frac{6}{2}
\]
\[
y = \frac{1}{2} x + \frac{3}{2}
\]
\[
y = \frac{1}{2} x + \frac{3}{2}
\]
The equations are equivalent. ■

Application: Writing a Linear Equation

Example 3  Corbin is buying concert tickets for some of his friends. The table shows the cost when purchasing different amounts of tickets. The total cost includes a parking fee of $12.

<table>
<thead>
<tr>
<th>Number of Tickets ((x))</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Cost ((y))</td>
<td>$29</td>
<td>$46</td>
<td>$63</td>
<td>$80</td>
</tr>
</tbody>
</table>

A. Write the equation that gives the total cost \( y \) in terms of the number of tickets purchased \( x \) in point-slope form.

B. Convert the equation from part A to slope-intercept form.

C. What is the meaning of the \( y \)-intercept?

D. What is the meaning of the slope?

Solution

A.

Step 1  Use the table to find the slope of the line. Choose any two data pairs in the table.

Let \((x_1, y_1) = (2, 29)\) and \((x_2, y_2) = (4, 46)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{46 - 29}{4 - 2} = \frac{17}{2}
\]
The slope of the line is \(\frac{17}{2}\).

Step 2  Write the equation in point-slope form, using the data point \((2, 29)\).

\[
y - y_1 = m(x - x_1)
\]
\[
y - 29 = \frac{17}{2} (x - 2)
\]
B. Write the equation in slope-intercept form.

\[ y - 29 = \frac{17}{2}(x - 2) \]

\[ y - 29 = \frac{17}{2}x - 17 \quad \text{Distributive Property} \]

\[ y = \frac{17}{2}x - 17 + 29 \quad \text{Add 29 to each side.} \]

\[ y = \frac{17}{2}x + 12 \quad \text{Simplify.} \]

C. The \( y \)-intercept represents the cost of parking.

D. The cost of each ticket is the slope of the line. Write \( \frac{17}{2} \) as a decimal.

\[ \frac{17}{2} = 8.5 \]. The cost of each ticket is $8.50. ■

Problem Set

Find the slope.

1. \( y - 2 = -3(x + 1) \)
2. \( y + 6 = \frac{1}{3}(x - 4) \)
3. \( y + 7 = -\frac{3}{4}(x - 4) \)

Use point-slope form to write the equation of the line that passes through the given point and has the given slope.

4. \((1, 2); m = -5\)
6. \((0, 5); m = \frac{1}{3}\)
8. \((-1, -4); m = 3\)
5. \((-3, 4); m = 7\)
7. \((0, 0); m = \frac{3}{4}\)
9. \((7, -8); m = \frac{1}{7}\)

For each problem:

A. Find the slope of the line.
B. Write two equations of the line in point-slope form—one for each of the points given on the graph.

10.

11.
Graph the line.

16. \( y - 2 = 3(x - 1) \)
17. \( y + 4 = \frac{1}{3}(x + 2) \)
18. \( y - 3 = -\frac{1}{2}(x + 1) \)
19. \( y + 2 = -4x \)
20. \( y = 2(x + 6) \)
21. \( y + 1 = \frac{1}{6}(x - 4) \)

Solve.

22. Mr. Gonzales drives a toll road every day to reach his job. The table below shows the total cost for the miles driven.

A. Write an equation that gives the total cost \( y \) in terms of the number of miles traveled \( x \).

B. How much will Mr. Gonzales pay if he drives 32 miles?

<table>
<thead>
<tr>
<th>Number of Miles</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>$7.60</td>
<td>$15.10</td>
<td>$22.60</td>
<td>$30.10</td>
</tr>
</tbody>
</table>

23. Soo-Min has been practicing her jumping for years. The table below shows her jumping height at four ages.

A. Write an equation that gives the total jumping height \( y \) in terms of her age \( x \).

B. How high should Soo-Min be able to jump at age \( 1\frac{1}{2} \) if she continues progressing at the same rate?

<table>
<thead>
<tr>
<th>Age</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jumping Height</td>
<td>3&quot;</td>
<td>5&quot;</td>
<td>7&quot;</td>
<td>9&quot;</td>
</tr>
</tbody>
</table>
24. Dweezil has a piggie bank with pennies and quarters. As he adds more quarters to the bank, he makes a table that shows the value of all of the coins in the bank given a specific number of quarters.

A. Write an equation that gives the total value of coins $y$ in terms of the number of quarters $x$.

B. How many pennies are in the bank?

C. What will the value of the collection be when the bank contains 20 quarters?

<table>
<thead>
<tr>
<th>Number of Quarters</th>
<th>5</th>
<th>8</th>
<th>11</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of Coins</td>
<td>$10.25</td>
<td>$11</td>
<td>$11.75</td>
<td>$12.5</td>
</tr>
</tbody>
</table>

25. Alonzo saves $50 a month from money earned at his job. His grandparents also give him $\frac{1}{2}$ of that amount each month. The table below shows his total savings.

A. Write an equation that gives his savings $y$ in terms of the number of months he works $x$.

B. How much will Alonzo have in his account after 2 years?

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>6</th>
<th>11</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Savings</td>
<td>$75</td>
<td>$325</td>
<td>$575</td>
<td>$825</td>
</tr>
</tbody>
</table>

26. The company NewDisks manufactures CDs. The table shows the total production cost $y$ for $x$ CDs which includes materials and labor.

A. Write an equation that gives the total production cost $y$ in terms of the number of CDs manufactured $x$.

B. Can the company manufacture 2000 CDs for under $130?

<table>
<thead>
<tr>
<th>Number of CDs</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production Cost</td>
<td>$15</td>
<td>$20</td>
<td>$25</td>
<td>$30</td>
</tr>
</tbody>
</table>

27. Lamar has started walking for exercise. His results for the last week are in the table below.

A. Write an equation that gives the time $y$ in terms of the distance walked $x$.

B. How long will it take Lamar to walk 10 miles?

<table>
<thead>
<tr>
<th>Distance (miles)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (minutes)</td>
<td>15</td>
<td>30</td>
<td>45</td>
<td>60</td>
</tr>
</tbody>
</table>

28. Farmer Cal plants corn every year. The table shows the total cost of producing a corn crop given the number of acres available.

A. Write an equation that gives the total cost $y$ in terms of the number of available acres $x$.

B. Can 1000 acres of corn be farmed for less than $1700?

<table>
<thead>
<tr>
<th>Number of Acres</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of Crop</td>
<td>$195</td>
<td>$370</td>
<td>$545</td>
<td>$720</td>
</tr>
</tbody>
</table>

29. **Challenge** The Hermitage Car Company recorded the sales of its new special edition sports car for one month. The results are in the table below.

A. Write an equation that gives the sales $y$ in terms of the number of available sports cars on the lot $x$.

B. Write in words what the data point (75, 5) means in this application.

C. As the number of sports cars on the lot increases what happens to demand?

<table>
<thead>
<tr>
<th>Number of Sports Cars on the Lot</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

30. **Challenge** The average price of gas increases each year as indicated in the table below.

A. Write an equation that gives the average cost in terms of the year $x$.

B. Find the missing values for years 2 and 4.

C. Assuming a constant rate of increase, what year would the average cost of gas be $4.60$?

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Cost</td>
<td>$2.20</td>
<td>$2.60</td>
<td>$3.00</td>
<td>$3.40</td>
</tr>
</tbody>
</table>
Parallel and Perpendicular Lines

You can use the slope of a line to determine characteristics of the line. You can also use the slopes of two (or more) lines to determine if the lines are parallel or perpendicular.

**PROPERTY**

Slopes of Parallel Lines
- If two lines have the same slope, then the lines are parallel.
- If two lines are parallel, then the lines have the same slope.

**REMEMBER**

Two lines that lie in the same plane are parallel if they never intersect.

---

**Writing the Equation of a Line Parallel to Another Line**

**Example 1** Find the equation of the line that passes through the point \((2, -1)\) and is parallel to the line \(8x + 2y = 10\). Write the equation of the parallel line in slope-intercept form.

**Solution**

**Step 1** Find the slope of the line \(8x + 2y = 10\). Start by writing the equation in slope-intercept form.

\[
8x + 2y = 10
\]

\[
2y = -8x + 10
\]

\[
y = -4x + 5
\]

The slope of the line is \(-4\).

**Step 2** The slope of the parallel line is \(-4\) and a point on the line is \((2, -1)\). Use point-slope form to find the equation of the line.

\[
y - y_1 = m(x - x_1)
\]

Use point-slope form.

\[
y - (-1) = -4(x - 2)
\]

Substitute \(-1\) for \(y_1\), \(-4\) for \(m\), and \(2\) for \(x_1\).

\[
y + 1 = -4x + 8
\]

Simplify.

\[
y + 1 - 1 = -4x + 8 - 1
\]

Subtract \(1\) from each side.

\[
y = -4x + 7
\]

Simplify.

The equation of the parallel line is \(y = -4x + 7\). □
Writing the Equation of a Line Perpendicular to Another Line

Example 2  Find the equation of the line that passes through the point (1, 3) and is perpendicular to the line $12x + y = 6$. Write the equation of the perpendicular line in slope-intercept form.

Solution

Step 1  Find the slope of the line $12x + y = 6$. Write the equation in slope-intercept form.

$12x + y = 6$

$y = -12x + 6$

The slope of the line is $-12$.

Step 2  The slope of the perpendicular line is the negative reciprocal of $-12$ which is $\frac{1}{12}$. Find the equation of the line that passes through the point (1, 3) and has a slope of $\frac{1}{12}$. Use point-slope form to find the equation of the line.

$y - y_1 = m(x - x_1)$  Use point-slope form.

$y - 3 = \frac{1}{12}(x - 1)$  Substitute 3 for $y_1$, $\frac{1}{12}$ for $m$, and 1 for $x_1$.

$y - 3 = \frac{1}{12}x - \frac{1}{12}$  Distributive Property

$y - 3 + 3 = \frac{1}{12}x - \frac{1}{12} + 3$  Add 3 to each side.

$y = \frac{1}{12}x - \frac{1}{12} + \frac{36}{12}$  Rename 3 as $\frac{36}{12}$.

$y = \frac{1}{12}x + \frac{35}{12}$  Simplify.

The equation of the perpendicular line is $y = \frac{1}{12}x + \frac{35}{12}$. ■

Determining Whether Lines are Parallel, Perpendicular, or Neither

To determine if lines are parallel or perpendicular, find the slope of each line. If the slopes are the same, the lines are parallel. If the slopes are negative reciprocals, the lines are perpendicular.
Example 3  Determine whether each pair of lines is parallel, perpendicular, or neither.

A.  \[4x + 3y = 1\]
    \[8x + 6y = 12\]

Solution  Write each line in slope-intercept form to identify its slope.

**Line 1:**
\[4x + 3y = 1\]
\[3y = -4x + 1\]
\[y = \frac{-4x + 1}{3}\]
\[y = -\frac{4}{3}x + \frac{1}{3}\]

**Line 2:**
\[8x + 6y = 12\]
\[6y = -8x + 12\]
\[y = -\frac{8x + 12}{6}\]
\[y = -\frac{4}{3}x + 2\]

The slope of both lines is \(-\frac{4}{3}\), so the lines \(4x + 3y = 1\) and \(8x + 6y = 12\) are parallel. ■

B.  \[y = -2x + 7\]
    \[2x + 5y = 10\]

Solution  

**Line 1:**
The line is in slope-intercept form. \[y = -2x + 7\]
The slope is \(-2\).

**Line 2:**
\[2x + 5y = 10\]
\[5y = -2x + 10\]
\[y = -\frac{2x + 10}{5}\]
\[y = -\frac{2}{5}x + 2\]

The slope of the line is \(-\frac{2}{5}\).

The slopes of the lines are different and are not negative reciprocals, so the lines \(y = -2x + 7\) and \(2x + 5y = 10\) are neither parallel nor perpendicular. ■

C.  \[y = -4x + 7\]
    \[x - 4y = 3\]

Solution  

**Line 1:**
\[y = -4x + 7\]
The slope of the line is \(-4\).

**Line 2:**
\[x - 4y = 3\]
\[-4y = -x + 3\]
\[y = \frac{1}{4}x - \frac{3}{4}\]

The slope of the line is \(\frac{1}{4}\). (continued)
The slopes of the lines are negative reciprocals of each other, so the lines $y = -4x + 7$ and $x - 4y = 3$ are perpendicular.

Application: City Map

Example 4  The diagram shows streets on a coordinate grid. Is Park Avenue parallel to Ohio Avenue?

Solution  Find the slope of each line using the slope formula.

*Park Avenue*:

Let $(x_1, y_1) = (3, 7)$ and $(x_2, y_2) = (1, 4)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 7}{1 - 3} = \frac{-3}{-2} = \frac{3}{2}$$

*Ohio Avenue*:

Let $(x_1, y_1) = (8, 3)$ and $(x_2, y_2) = (6, 0)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{6 - 8} = \frac{-3}{-2} = \frac{3}{2}$$

The slopes are equal, so Park Avenue is parallel to Ohio Avenue.

Problem Set

For each problem:

A. Find the slope of the given line.
B. Find the slope of the line that passes through the given point and is parallel or perpendicular to the given line.
C. Write the equation of the line from part B in slope-intercept form.

1. (2, 3), parallel to $4x - 2y = 8$
2. (−1, 5), parallel to $3x + 2y = 4$
3. (4, 7), perpendicular to $6x + 3y = 12$
4. (−2, 15), perpendicular to $2x - 3y = 4$
5. (−4, −2), parallel to $10x - 5y = 15$
6. (7, 0), perpendicular to $x + y = 6$
7. (−1, 1), perpendicular to $6x - 7y = -8$
8. (0, 6), parallel to $y = 7x + 12$
9. (−2, −9), perpendicular to $24x + 3y = 0$
10. (3, −8), parallel to $x + y = 10$
11. (3, −4), perpendicular to −40x − 8y = 16
12. (0, 0), parallel to −4x − 6y = 9
13. (8, 1), perpendicular to x = 20
14. (−9, 0), parallel to −3x + 3y = 11

Determine whether each pair of lines is parallel, perpendicular, or neither.

18. \(2x − y = −1\)
   \(4x − 2y = 6\)
19. \(y = \frac{1}{3}x + 6\)
   \(2x − 3y = 12\)
20. \(x + 10y = −20\)
   \(y = 10x − 1\)
21. \(7x − y = −5\)
   \(14x − 2y = 18\)

22. \(y = 4x + 1\)
   \(4x + 16y = 80\)
23. \(y = 5x − 4\)
   \(15x − 3y = −9\)

24.

25.

26.
27. Solve.

28. Solve.

*29. Challenge Bing, Plaid, Croon, and Nod are parallel streets in the city. A new street called Duncan is constructed to run perpendicular to the four streets. Find the equation of Duncan street if it intersects Nod street at the point (6, 4). Write the equation in slope-intercept form.

*30. Challenge A square is formed by four lines. Opposite sides of the square are parallel and adjacent sides are perpendicular. Using the slope formula, find the y-coordinate of point C so that figure ABCD is a square.
Equations from Graphs

You know how to graph a line when given an equation. You will now use the graph of a line to determine the equation of the line.

Writing the Equation of a Line in Point-Slope Form
Using a Graph

When you know two points that lie on a line, you can write the equation of the line using point-slope form.

Example 1  Write the equation of the line shown in the graph.

Solution

Step 1  Use the slope formula to find the slope of the line.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-1)}{8 - 2} = \frac{3}{6} = \frac{1}{2} \]

Step 2  Write the equation in point-slope form. You can use either point in the point-slope equation. Using the point \((8, 2)\):

\[ y - y_1 = m(x - x_1) \quad \text{Point-slope form} \]

\[ y - 2 = \frac{1}{2}(x - 8) \quad \text{Substitute } 2 \text{ for } y_1, \frac{1}{2} \text{ for } m, \text{ and } 8 \text{ for } x_1. \]

The equation of the line in point-slope form is \( y - 2 = \frac{1}{2}(x - 8). \) ■
**Using a Graph to Write the Equation of a Line in Standard Form**

**Example 2**  Write the equation of the line from Example 1 in standard form.

**Solution**  Rewrite the equation \( y - 2 = \frac{1}{2}(x - 8) \) in standard form.

\[
y - 2 = \frac{1}{2}(x - 8) \quad \text{Equation in point-slope form}
\]

\[
y - 2 = \frac{1}{2}x - 4 \quad \text{Distributive Property}
\]

\[
y = \frac{1}{2}x - 2 \quad \text{Add 2 to each side of the equation.}
\]

\[
-\frac{1}{2}x + y = -2 \quad \text{Subtract \( \frac{1}{2}x \) from each side.}
\]

\[
2\left(-\frac{1}{2}x + y\right) = 2(-2) \quad \text{Multiple each side by 2.}
\]

\[
-x + 2y = -4 \quad \text{Simplify.}
\]

The equation of the line in standard form is \(-x + 2y = -4\). ■

**Using a Graph to Write the Equation of a Line in Slope-Intercept Form**

When you know two points that lie on a line and one of those points is the \( y \)-intercept, you can write the equation of the line using slope-intercept form.

**Example 3**  Write the equation of the line shown in the graph.

**Solution**

**Step 1**  Use the slope formula to find the slope of the line.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-2)}{-3 - 0} = \frac{9}{-3} = -3
\]
Step 2  Write the equation in slope-intercept form. The line intersects the y-axis at the point (0, –2) so the y-intercept is –2.

\[ y = mx + b \]  
Slope-intercept form

\[ y = -3x + (-2) \]  
Substitute –3 for \( m \) and –2 for \( b \).

\[ y = -3x - 2 \]  
Simplify.

The equation of the line in slope-intercept form is \( y = -3x - 2 \).

Writing the Equation of a Horizontal and a Vertical Line

Example 4  Write the equation of each line shown in the graph.

A. the horizontal line

Another way to solve the problem is to find the slope of the line and use slope-intercept form to write the equation of the line. The line crosses the y-axis at (0, –5) so the y-intercept is –5.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - (-5)}{1 - (-4)} = \frac{0}{5} = 0 \]

\[ y = mx + b \]

\[ y = 0 \cdot x + (-5) \]

\[ y = -5 \]  

B. the vertical line

The x-coordinate of every point on the vertical line is 3. The equation of the line is \( x = 3 \).

REMEMBER

The slope of a vertical line is undefined, so you can’t use the point-slope or slope-intercept formulas to find the equation.
Problem Set

Write the equation of the line in point-slope form.

1. \( y = \frac{x}{2} + 2 \)

2. \( y = -x + 8 \)

3. \( y = x - 6 \)

4. \( y = -\frac{x}{2} + 5 \)

5. \( y = -\frac{x}{2} + 4 \)

6. \( y = \frac{x}{2} - 4 \)

7. \( y = -x + 2 \)

8. \( y = -\frac{x}{2} - 5 \)
Use point-slope form to write the equation of the line in standard form.

9. 

10. 

11. 

12. 

13. 

14. 

15. 

16.
Write the equation of the line in slope-intercept form.

17. 

18. 

19. 

20. 

21. 

22. 

23. 

24. 

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Applications: Linear Models

Use a table of values to help you understand and solve application problems involving constant rates.

**Distance**

Example 1 The distance $d$ that Cosimo can walk is given by the linear equation $d = 3.5t$, where $t$ is the time in hours. Make a table showing the values for $d$ when $t = 1, 2, 3, \text{ and } 4$. Then graph the linear equation.

**Solution**

**Step 1** Make a table. Substitute each value of $t$ into the equation and solve for $d$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>$d = 3.5t$</td>
<td>$d = 3.5t$</td>
<td>$d = 3.5t$</td>
<td>$d = 3.5t$</td>
</tr>
<tr>
<td></td>
<td>$= 3.5 \cdot 1$</td>
<td>$= 3.5 \cdot 2$</td>
<td>$= 3.5 \cdot 3$</td>
<td>$= 3.5 \cdot 4$</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>7</td>
<td>10.5</td>
<td>14</td>
</tr>
</tbody>
</table>

**Step 2** Plot the ordered pairs $(1, 3.5), (2, 7), (3, 10.5), \text{ and } (4, 14)$. Then draw the line. Since distance and time cannot be negative, the graph of the line is restricted to quadrant I.
Cost

Example 2  An online movie rental company charges customers $5 to sign up and a monthly fee of $10. The total cost $C$ for $n$ months can be modeled by the equation $C = 10n + 5$.

A. Complete a table of values. What is the total cost for 1 month? 5 months? 7 months? 1 year?

B. Is this a linear model? Explain.

Solution

A. Substitute each value of $n$ into the equation and solve for $C$.

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>5</th>
<th>7</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>$10n + 5$</td>
<td>$10n + 5$</td>
<td>$10n + 5$</td>
<td>$10n + 5$</td>
</tr>
<tr>
<td></td>
<td>$10 \cdot 1 + 5$</td>
<td>$10 \cdot 5 + 5$</td>
<td>$10 \cdot 7 + 5$</td>
<td>$10 \cdot 12 + 5$</td>
</tr>
<tr>
<td></td>
<td>$10 + 5$</td>
<td>$50 + 5$</td>
<td>$70 + 5$</td>
<td>$120 + 5$</td>
</tr>
<tr>
<td></td>
<td>$15$</td>
<td>$55$</td>
<td>$75$</td>
<td>$125$</td>
</tr>
</tbody>
</table>

When $n$ is 1, $C$ is 15, so the total cost for 1 month is $15. When $n$ is 5, $C$ is 55, so the total cost for 5 months is $55. When $n$ is 7, $C$ is 75, so the total cost for 7 months is $75. The variable $n$ is time in months, so change 1 year to 12 months. When $n$ is 12, $C$ is 125, so the total cost for 12 months is $125.

B. Examine the rate of change between values.

(1, 15) and (5, 55) rate of change $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{55 - 15}{5 - 1} = \frac{40}{4} = 10$

(5, 55) and (7, 75) rate of change $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{75 - 55}{7 - 5} = \frac{20}{2} = 10$

(7, 75) and (12, 125) rate of change $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{125 - 75}{12 - 7} = \frac{50}{5} = 10$

The rates of change are equal. Therefore, the model is linear. Another way to determine if the model is linear is to look at the equation $C = 10n + 5$. Notice the equation is in slope-intercept form where the slope of the line is 10 and the y-intercept is 5. Only linear equations are written in slope-intercept form.

Problem Set

For each problem:

A. Make a table of values.
B. Graph the linear equation.

1. The time $t$ that it takes Hal to read a lesson and complete the practice problems is given by the linear equation $t = 30 + 3p$, where $p$ is the number of practice problems. Determine the time when $p = 5$, 10, 15, and 20.

2. Jacob is mowing grass for the summer and his fee is given by the linear equation $f = 0.01a + 10$, where $a$ is the yard area in square feet. Determine the fee when $a = 500$, 1000, 1500, and 2000.
3. Stephania is ordering sandwiches for a luncheon and to determine the number of sandwiches that she must order, she uses the linear equation 
\[ s = 3g + 3 \], where \( g \) is the number of guests she expects to join her. Determine the number of sandwiches when \( g = 0, 2, 4, \) and 6.

4. Hao bought a membership at a golf course, and the price he pays per month to golf is shown by the linear equation 
\[ p = 30 + 20r \], where \( r \) is the number of rounds of golf he plays. Determine the price when \( r = 0, 1, 2, \) and 3.

5. For a fundraiser, elementary school students are selling a $5 discount card for a local restaurant. The savings from buying and using the card is shown by the linear equation 
\[ s = 5d - 25 \], where \( d \) is the number of dinners purchased at the restaurant with the card. Determine the savings when \( d = 0, 3, 6, \) and 9.

6. A computer salesman is paid a base salary plus commission for each computer sold. Her hourly wage is shown by the linear equation 
\[ w = 5 + 2c \], where \( c \) is the number of computers sold in a given hour. Determine the wage when \( c = 1, 2, 3, \) and 4.

For each problem:

A. Make a table using reasonable input values.

B. Is this a linear model? Explain. If so, give the slope and \( y \)-intercept.

11. During deceleration, a car’s speed can be shown by the equation 
\[ s = 35 - 2t^2 \], where \( t \) is the time in seconds that the car decelerated.

12. Two years after the purchase, if Manuel’s gift card is unused, the automatic decrease on Manuel’s gift card balance is given by the equation 
\[ b = 50 - 5m \], where \( m \) is the number of months beyond two years.

13. The number of customers that come to a fruit stand during the week can be modeled by the equation 
\[ c = 0.3x \], where \( x \) is the temperature in degrees Fahrenheit.

14. The total deposit and rent collected by an apartment manager for one resident are given by the equation 
\[ t = 350 + 700m \], where \( m \) is the number of months the resident has lived there.

15. At a local restaurant, the number of meal orders that can be filled in five minutes is given by the equation 
\[ m = -(n - 4)^2 + 10 \], where \( n \) is the number of employees working in the kitchen.

16. The total amount in Sakura’s savings account is given by the equation 
\[ P = 500 \cdot (1.1)^{t} \], where \( t \) is the number of years the money is in the account.

17. The total amount Ebony paid for house paint is given by the equation 
\[ T = 24.99c + 0.07 \cdot 24.99c \], where \( c \) is the number of cans of paint that she bought.

18. The amount Leon pays for electricity each month is given by the equation 
\[ E = 9.99 + \frac{11.07w}{100} + 4.99 \], where \( w \) is the number of kilowatt-hours he uses. For the table, show the cost of electricity when \( w = 700, 800, 900, \) and 1000.
19. A car salesman’s daily pay is given by the equation \( S = 48 + 0.005v \), where \( v \) is the value of the cars he sold that day. For the table, show values for \( S \) when \( v = 15,000, 20,000, 25,000, \) and \( 30,000 \).

20. The amount of a 400 g sample of a radioactive substance that remains over time is given by the equation \( A = 400 \cdot \left( \frac{1}{4} \right)^d \), where \( d \) is the number of days since the sample weight was 400 g.

For each problem:
A. Make a table using the given input values.
B. Is this a linear model? Explain using rates of change.

21. At its top speed, the distance a cheetah can run in meters is given by the equation \( d = 1850t \), where \( t \) is the number of minutes. Determine the distance when \( t = 1, 2, 3, \) and \( 4 \).

22. Hannah charges an hourly babysitting fee of $6 for two children, $7 for three children, and $8 for four or five children.

23. The monthly fee at Teresa’s gym is given by the equation \( f = 30 + 15c \), where \( c \) is the number of fitness classes for which she is registered. Determine the fee when \( c = 0, 1, 2, \) and \( 3 \).

24. At a predicted growth rate of 0.9%, Canada’s population in millions of people is given by the equation \( P = 33(1 + 0.009)^t \), where \( t \) is the number of years since 2006. Determine the population, in millions, for \( t = 5, 10, 15, \) and \( 20 \).

25. The total cost \( T \) of admission to the zoo is given by the equation \( T = 13.50p \), where \( p \) is the number of people. Determine the cost when \( p = 2, 4, 6, \) and \( 8 \).

26. The amount Audrey pays to order digital photos online is given by the equation \( A = 0.2p + 5 \), where \( p \) is the number of photos she orders. Determine the amount paid when \( p = 5, 10, 15, \) and \( 20 \).

27. An online distributor sells couscous for $2.99 per pound and charges a shipping fee of $1.50 per pound. Determine the total amount paid \( A \) when the number of pounds purchased \( p \) is 1, 3, 5, and 7.

Solve.

*28. Challenge* An auto club membership provides 3 free oil changes, then charges $20 per oil change after that.

A. Make a table to show the amount paid as a function of the total number of oil changes for 1, 2, 3, 4, and 5 oil changes.

B. Explain how this problem involves linear models and define them and the ranges in which they are valid.

*29. Challenge* Asher works in a stockroom and gets paid $8 per hour for up to 40 hours per week. He gets paid $12 per hour for any additional hours that week.

A. Make a table to show his pay as a function of hours worked for 20, 30, 40, 50, and 60 hours in a week.

B. Explain how this problem involves linear models and define them and the ranges in which they are valid.
Graphing Linear Inequalities

You know how to graph linear equations. It is also possible to graph linear inequalities.

Solutions to Linear Inequalities in Two Variables

An ordered pair \((x, y)\) is a solution to an inequality in two variables if substituting the values of \(x\) and \(y\) into the inequality yields a true statement.

Example 1  Determine if each ordered pair is a solution to the inequality \(y < 4x - 8\).

A.  \((0, 0)\)  B.  \((2, -10)\)

Solution  Test each ordered pair. Substitute the values of \(x\) and \(y\) into the inequality \(y < 4x - 8\). Then simplify. If the resulting statement is true, the ordered pair is a solution.

A.  \((0, 0)\)

\[
y < 4x - 8 \quad 0 < 4 \cdot 0 - 8 \quad 0 \not< -8
\]

(0, 0) is not a solution to the inequality \(y < 4x - 8\).

B.  \((2, -10)\)

\[
y < 4x - 8 \quad -10 < 4 \cdot 2 - 8 \quad -10 < 0 \checkmark
\]

(2, -10) is a solution to the inequality \(y < 4x - 8\).

Graphing a Linear Inequality in Two Variables

The graph of a linear inequality is a region of the coordinate plane called a half-plane.

A boundary line divides the coordinate plane into two half-planes. One of the half-planes contains all the points that are solutions to the inequality. The boundary line is dashed if the boundary is not part of the solution, so the inequality uses the operator \(<\) or \(>\). The boundary is a solid line if it is part of the solution, so it uses the operator \(\leq\) or \(\geq\).

(continued)
To graph a linear inequality in two variables, first graph the boundary line. Then determine which half-plane contains the solutions to the inequality and shade that half-plane. The shaded region represents all the solutions to the inequality.

**GRAPHING A LINEAR INEQUALITY**

**Step 1** Graph the boundary line. If the boundary is included in the solution, use a solid line. If the boundary is not included in the solution, use a dashed line.

**Step 2** Test a point that does not lie on the boundary line to determine if the ordered pair is a solution to the inequality.

**Step 3** If the ordered pair is a solution, shade the half-plane that contains the point. Otherwise shade the other half-plane.

**Example 2** Graph the inequality.

A. \( y > 2x - 1 \)

**Solution**

**Step 1** Graph the boundary line \( y = 2x - 1 \). Since the inequality contains the operator \( > \) the boundary line is a dashed line.

**Step 2** Test a point that does not lie on the boundary line to determine if the ordered pair is a solution to the inequality. Test the point \((0, 0)\).

\[
y > 2x - 1 \\
0 > 2 \cdot 0 - 1 \\
0 > -1 \checkmark \\
0 > -1 \text{ is a true statement.}
\]

**Step 3** Since \((0, 0)\) is a solution, shade the half-plane that contains the point \((0, 0)\). ■
B. \[ 3x - 4y \geq 8 \]

**Solution**

**Step 1** To graph the boundary line, first write the inequality in slope-intercept form.

\[ 3x - 4y \geq 8 \]

\[ -4y \geq -3x + 8 \]

Subtract 3x from each side of the inequality.

\[ \frac{-4y}{-4} \leq \frac{-3x + 8}{-4} \]

Divide each side of the inequality by -4. Since you are dividing an inequality by a negative number, switch the direction of the inequality.

\[ y \leq \frac{3}{4}x - 2 \]

Simplify.

Now, graph the boundary line \( y = \frac{3}{4}x - 2 \). Since the inequality contains the operator \( \leq \), the boundary line is a solid line.

**Step 2** Test a point that does not lie on the boundary line to determine if the ordered pair is a solution to the inequality. Test the point (0, 0).

\[ y \leq \frac{3}{4}x - 2 \]

\[ 0 \leq \frac{3}{4} \cdot 0 - 2 \]

\[ 0 \leq -2 \]

0 \( \leq \) -2 is a false statement.

**Step 3** Since (0, 0) is not a solution, shade the half-plane that does not contain the point (0, 0).

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**Application: Mixture**

**Example 3** Audrina wants to create a blend of birdseed containing millet seed and sunflower seed. Millet seed costs $3.50 per pound and sunflower seed costs $2 per pound. She has $28 to spend on the birdseed.

A. Write an inequality to model the problem.

B. Graph the inequality.

C. What does the shaded area of the graph represent?

**Solution**

A. Write an inequality which can be used to model the problem.

<table>
<thead>
<tr>
<th>Cost of a Pound of Millet Seed</th>
<th>( \times )</th>
<th>Number of Pounds of Millet Seed Purchased</th>
<th>+</th>
<th>Cost of a Pound of Sunflower Seed</th>
<th>( \times )</th>
<th>Number of Pounds of Sunflower Seed Purchased</th>
<th>( \leq )</th>
<th>Amount of Money Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.50</td>
<td>( x )</td>
<td>+</td>
<td></td>
<td>$2</td>
<td>( y )</td>
<td>( \leq )</td>
<td>$28</td>
<td></td>
</tr>
</tbody>
</table>

The problem is modeled by the inequality \( 3.5x + 2y \leq 28 \).
Problem Set

Determine if each ordered pair is a solution to the given inequality.

1. \( y \geq 1 - x \)
   A. (4, 3)
   B. (−2, 8)

2. \( y < 0.5x + 9 \)
   A. (−7, 1)
   B. (−6, 6)

3. \( 2y - 3x > 12 \)
   A. (−4, −17)
   B. (12, 3)

4. \( \frac{1}{4}x \leq \frac{7}{3}y + 2 \)
   A. (48, 9)
   B. (36, −3)

5. \( y \geq 11x - 119 \)
   A. (9, 20)
   B. (−9, −20)

6. \( 20y > 15 - 50x \)
   A. (−0.5, 0.75)
   B. (1.2, −0.25)

7. \( -y \leq -4 + x \)
   A. (13, −11)
   B. (−7, 3)

8. \( x + 3y < -17 \)
   A. (−1, −5)
   B. (−29, 4)

9. \( 2y > 14 - 7x \)
   A. \( \left(\frac{2}{7}, 7\right) \)
   B. \( \left(-\frac{4}{7}, 9\right) \)

Graph the inequality.

10. \( y \leq 10 - 3x \)

11. \( y - 2x > -8 \)

12. \( x < 17 + y \)

13. \( 2y \geq 6x + 6 \)

14. \( 5x + 4y < 2 \)

15. \( x - y \geq 70 \)

16. \( -5y < 10 + x \)

17. \( 2y + 1 \geq -3 - \frac{3}{2}x \)

18. \( y \leq 6x - 13 \)

19. \( y - 0.25 < 0.5x \)

20. \( 8x \geq 12 - 2y \)

21. \( y < 19 - 6x \)

22. \( 3 - \frac{1}{3}y \leq \frac{2}{9}x \)

23. \( y - x > 50 \)
Solve. For each problem:
A. Write an inequality to model the problem.
B. Graph the inequality.
C. Explain any restrictions on the graph.

24. Ciara budgeted $30 per month for entertainment. Movies cost $7 and sports games cost $5.

25. A company prepares a graph of its profit forecast. The annual budget predicts monthly income of at least $5000 more than expenditures.

26. Ameesh wants to calculate the amount of fuel used by his car on the highway versus the amount of fuel used by his car on back roads over the same distance. He knows that his car uses less than twice as many gallons of fuel per mile on the back roads as it uses on the highway.

27. Neyreich wants to arrange a bouquet of roses and tulips that is worth at least $20. Roses cost $5 each and tulips cost $4 each.

28. A salad dressing recipe requires at least 8 ounces of oil to be combined with some combination of vinegar and lemon juice in a 16-ounce container.

29. Challenge Braeden has a bag of 200 coins. More than 50 of them are quarters and at least 75 of them are dimes. How many of the coins are quarters or dimes?

30. Challenge A bean soup mix is made from lentils, which cost $4 per pound, kidney beans, which cost $3 per pound, and navy beans and black-eyed peas, which each cost $10 per pound. The soup mix has half as many navy beans as kidney beans and twice as many black-eyed peas as kidney beans. What combination of lentils and kidney beans can be used to make a soup mix that costs less than $20?
Inequalities from Graphs

Just as you can use the graph of a line to determine the equation of the line, you can use the graph of an inequality to write the inequality.

Writing an Inequality Given a Graph

Example 1  Write the inequality shown in the graph.

Solution

Step 1  Find the equation of the boundary line.

You know several methods for finding the equation of a line from a graph. Since you know the y-intercept of the line, use slope-intercept form. First find the slope of the line using the slope formula.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{3 - 0} = \frac{3}{3} = 1 \]

Write the equation in slope-intercept form. The line intersects the y-axis at the point (0, 1) so the y-intercept is 1.

\[ y = mx + b \quad \text{Slope-intercept form} \]

\[ y = 1x + 1 \quad \text{Substitute 1 for } m \text{ and } 1 \text{ for } b. \]

\[ y = x + 1 \quad \text{Simplify.} \]

The equation of the boundary line is \( y = x + 1 \).
Step 2  Choose a point in the half-plane that contains the solutions of the inequality. Substitute the x- and y-coordinates of the point into the equation of the boundary line, replacing the equal sign with an inequality operator that makes a true statement.

Use the point (4, 0).

\[ \begin{align*} y &= x + 1 \\ 0 &= 4 + 1 & \text{Substitute 4 for } x \text{ and 0 for } y. \\ 0 &= 5 & \text{Simplify.} \\ 0 &< 5 & \text{The less than operator makes the statement true.} \\ & & \text{Since the boundary line is a dashed line, the operator is } <. \\
& & \text{The inequality is } y < x + 1. \]

**Writing an Inequality in One Variable From a Graph**

**Example 2**  Write the inequalities shown in each graph.

**A.**

**Solution**

A. The equation of the boundary line is \( x = -4 \).

Test the point (0, 0).

\[ 0 \geq -4 \]

Since the boundary line is solid the operator is \( \geq \). The inequality is \( x \geq -4 \). ■

**B.**

The equation of the boundary line is \( y = 2 \).

Test the point (0, 0).

\[ 0 \leq 2 \]

Since the boundary line is solid the operator is \( \leq \). The inequality is \( y \leq 2 \). ■
Problem Set

Write the inequality shown in the graph.

1.

2.

3.

4.

5.

6.

7.

8.
Using the graphs below, determine the operator for a matching inequality of the form \( y \geq mx + b \). Explain.

*23. Challenge

*24. Challenge